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Regular and black-hole solutions of the Einstein-Yang-Mills-Higgs equations; the case of nonminimal coupling

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Abstract

Regular and black-hole solutions of the spontaneously broken Einstein-Yang-Mills-Higgs theory with nonminimal coupling to gravity are shown to exist. The main characteristics of the solutions are presented and differences with respect to the minimally coupled case are studied. Since negative energy densities are found to be possible, traversable wormhole solutions might exist. We prove that they are absent.

1 Introduction

In the light of the no-hair [1, 2] and no-go theorems [3, 4] for the classical glueball solutions with or without gravity, the discovery of both smooth and black-hole solutions of the self-gravitating non-Abelian gauge theories was a big surprise (for a review see [5]). Among such solutions a physically interesting case is that of a spontaneous broken EYMH theory examined in [6], where both regular and black-hole solutions, i.e. gravitating sphalerons and sphaleron black-holes have been found. The stability analysis of this system has shown that the solutions are unstable [7, 8].

Because of the physical importance of these objects, it is worthwhile to study generalizations of the couplings of the flat-space Lagangian to gravity. One of the simplest and best motivated extensions is the inclusion of an explicit coupling between the scalar field Φ and the curvature of the spacetime $\mathcal{R}$ of the form $\xi\Phi^{\dagger}\Phi\mathcal{R}$, where ξ is a dimensionless coupling constant. There are many reasons to believe that a nonminimal coupling term appears. A nonminimal coupling is generated by quantum corrections even if it is absent in the classical action and is required in order to renormalize the theory [9].

In many physical situations, inclusion of a $\xi \neq 0$ term leads to new interesting physical effects even at the classical level. Examples are the Bronnikov-Melnikov-Bacharova-Bekenstein conformal scalar hair [10, 11], the inflationary scenario with a nonminimally coupled "inflaton" field [12, 13], and boson star solutions [14]. For a review of the present situation see [15]. Two cases occur most frequently in the literature: "minimal coupling" (ξ=0) and "conformal coupling" (ξ=1/6). The conformal invariance dictates ξ=1/6 for a massless scalar field [9], while Nambu-Goldstone bosons have a minimal coupling ξ=0 [16]. However, there is no preferential value of ξ for a Higgs field in a unified gauge theory of electroweak interactions.

In this paper we study numerically regular and black-hole solutions of the coupled EYMH field equations with a nonminimal coupling to gravity, extending the results of ref.[6] to this case. Ref. [6] presented strong numerical arguments for the existence of both regular and black-hole solutions in a minimally coupled EYMH theory. For each fixed value of the Higgs vacuum expectation value v, solutions have been found, that can be indexed by the number of nodes k of the Yang-Mills potential function. For each k there are two branches of solution, depending on the behavior of $v \to 0$. For example, the so-called quasi $k = 0$ branch of solutions approaches the Schwarzschild solution as $v \to 0$; whereas the regular $k = 1$ branch approaches the first colored black-hole solution of the Einstein-Yang-Mills system [17] in the same limit. The two branches of solutions converge for some values of the theory parameters [7].

Although most of the phenomena discussed in [6] for the ξ=0 case repeat themselves in the general case, there are some important differences. For a nonminimal coupling, the time component of $T_{\mu\nu}$, which in Einstein's gravity would correspond to the local energy density, may be non-positive. Indeed, as we shall see later on, there are regions in space, where this quantity is negative. The reason is that, as a result of the nonminimal coupling with gravity, there

are contributions to $T_{\mu\nu}$ from the gravitational field itself . However, the local energy densities do not directly determine the sign of the asymptotic ADM mass, which is found to be positive.

Also, the parameter range of the solutions found in ref.[6] remains no longer valid and a new range has to be found for every choice of ξ. The existence of a nonminimal coupling between the Higgs field and the gravitational field implies a decrease of the maximal allowed vacuum expectation value of the Higgs field.

The paper is structured as follows: in section II we present the general framework and an analysis of the field equations, while in section III we adress the problem of the numerical construction of solutions. In section IV the possibility of the existence of Lorentzian wormholes is considered with a negative result. We conclude with section V where the results are compiled.

2 GENERAL FRAMEWORK AND BASIC EQUATIONS

Our study of the EYMH system is based upon the action

$$S=\int d^4x\sqrt{-g}[\frac{\mathcal{R}}{16\pi G}-\frac{1}{4\pi}((D_\mu\Phi)^\dagger(D^\mu\Phi)+V(\Phi)+\xi\Phi^\dagger\Phi\mathcal{R})-\frac{1}{4\pi}\frac{1}{4}\mid F\mid^2]. \quad (1)$$

Here G is the gravitational constant, D_μ is the usual gauge-covariant derivative expressed in the anti-hermitian basis of SU(2) ($\tau_a=-i\sigma_a/2$)

$$D_\mu=\partial_\mu+g\tau\cdot A_\mu, \quad (2)$$

g is the gauge coupling constant. Following [6], we assume that Φ possesses only one degree of freedom

$$\Phi=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \phi(x)\end{pmatrix}, \quad (3)$$

with ϕ real and time independent, and with the Higgs potential

$$V(\phi)=\frac{\lambda}{4}(\phi^2-v^2)^2, \quad (4)$$

where v denotes the vacuum expectation value of Φ. The action (1) becomes

$$S=\int d^4x\sqrt{-g}[\frac{\mathcal{R}}{16\pi G}-\frac{1}{4\pi}\frac{1}{2}((\partial_\mu\phi)(\partial^\mu\phi)+(\frac{g\phi}{2})^2A_\mu A^\mu+V(\phi)+\xi\phi^2\mathcal{R})-\frac{1}{4\pi}\frac{1}{4}\mid F\mid^2]. \quad (5)$$

There are considerable modifications in the Einstein equations due to the new energy-momentum tensor:

$$\begin{aligned} 8\pi T_{\mu\nu} = & \quad 8\pi T^{(minimal)}_{\mu\nu}+2\xi(G_{\mu\nu}\phi^2+g_{\mu\nu}\nabla_\gamma\nabla^\gamma\phi^2-\phi^2_{,\mu;\nu}) \quad (6)\\ 8\pi T^{(minimal)}_{\mu\nu} = & \quad 2F_{\mu\gamma}F^\gamma_\nu-\frac{1}{2}g_{\mu\nu}\mid F\mid^2+2(\frac{g\phi}{2})^2A_\mu A_\nu-g_{\mu\nu}(\frac{g\phi}{2})^2A_\gamma A^\gamma\\ & +2(\partial_\mu\phi)(\partial_\nu\phi)-g_{\mu\nu}((\partial_\gamma\phi)(\partial^\gamma\phi)+2V(\phi)). \quad (7)\end{aligned}$$

where $G_{\mu\nu}$ is the Einstein tensor. As we assume spherical symmetry it is convenient to use the usual metric form:

$$ds^2 = R^2(r)dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2) - \frac{dt^2}{T^2(r)} \tag{8}$$

where $R(r) = (1 - 2m(r)/r)^{-1/2}$ and $m(r)$ may be interpreted as the total mass-energy within the radius r. To describe the black-hole solutions we define $\delta = -ln(R/T)$; thus:

$$ds^2 = (1 - \frac{2m(r)}{r})^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2) - (1 - \frac{2m(r)}{r})e^{-2\delta(r)}dt^2. \tag{9}$$

The event horizon is at $r = r_h$ where $1/R^2(r_h) = 0$. In case there are several such zeroes, the horizon corresponds to the outer one. Regularity at the origin is satisfied when $T(0) < \infty$ and $R'(0) = T'(0) = 0$, while regularity at the event horizon $r=r_h$ requires $\delta(r_h) < \infty$. In this paper we deal with nonextremal black-holes only, i.e. near the event horizon

$$1 - 2m(r)/r \sim r - r_h. \tag{10}$$

A suitable rescaling of the time coordinate t implies:

$$R(0) = 1, \quad T(0) = 1, \quad m(r_h) = r_h/2, \quad \delta(r_h) = 0. \tag{11}$$

For the Yang-Mills field, it is convenient to use the ansatz discussed in [6]; thus a suitable parametrization of the Yang-Mills connection is:

$$A = \frac{1}{g}(1 + \omega)[-\widehat{\tau}_\varphi d\theta + \widehat{\tau}_\theta \sin\theta d\varphi]. \tag{12}$$

The $\widehat{\tau}_i$ are appropriately normalised spherical generators of the SU(2) group in the notation of ref. [6], *e.g.* $\widehat{\tau}_r = \widehat{r} \cdot \tau$, $[\tau_a, \tau_b] = \epsilon_{abc}\tau_c$, while $\phi(r)$ is the Higgs field.

Expressing the curvature scalar $\mathcal{R}$ in terms of the metric function $R(r)$ and $T(r)$, we obtain the following expression of the reduced action of our static spherically symmetric system:

$$S = \int drdt[\frac{1}{2G}\frac{1}{T}(R - \frac{1}{R} + 2r\frac{R'}{R}) - \frac{1}{2}(\frac{(\phi')^2 r^2}{RT} + \frac{\phi^2(1+\omega)^2}{2}\frac{R}{T})$$
$$-V(\phi)r^2\frac{R}{T} - \frac{1}{g^2}(\frac{(\omega')^2}{RT} + \frac{(1-\omega^2)^2}{2r^2}\frac{R}{T}) - \xi\frac{\phi^2}{RT}(R^2 - 1 + 2r\frac{R'}{R}) + 2\xi\phi\phi'\frac{r^2T'}{RT^2}] \tag{13}$$

for a regular spacetime, while a suitable form of the reduced action for a black-hole spacetime is

$$S = \int drdte^{-\delta}[\frac{m'(1 - 2\xi G\phi^2)}{G} - \frac{1}{2}((\phi')^2r^2(1 - \frac{2m}{r}) + \frac{\phi^2(1+\omega)^2}{2})$$
$$-V(\phi)r^2 - \frac{1}{g^2}((\omega')^2(1 - \frac{2m}{r}) + \frac{(1-\omega^2)^2}{2r^2}) + 2\xi\phi\phi'(r^2\delta'(1 - \frac{2m}{r}) + m'r - m)] \tag{14}$$

where the prime denotes derivative with respect to r.

A usual rescaling [18]

$$r \to rg/\sqrt{G}, \quad \phi \to \phi/\sqrt{G} \tag{15}$$

reveals that the solutions depend essentially on two dimensionless parameters α and β, expressible through the mass ratios

$$\alpha = \frac{M_W}{vM_{Pl}}; \quad \beta = \frac{M_H}{M_W} \tag{16}$$

with $M_W = gv$, $M_H = \sqrt{\lambda}v$ and $M_{Pl} = \frac{1}{\sqrt{G}}$. $V(\phi) = \frac{\beta^2}{4}(\phi^2 - \alpha^2)^2$ is the standard double well field potential. The field equations imply the relations

$$\omega'' = \omega'(\frac{R}{R'} + \frac{T}{T'}) + \frac{\omega(\omega^2-1)}{r^2}R^2 + \frac{\phi^2(\omega+1)}{4}R^2 \tag{17}$$

for the gauge field, and

$$\phi'' = \phi'(\frac{R}{R'} + \frac{T}{T'} - \frac{2}{r}) + R^2[\frac{\phi(1+\omega)^2}{2r^2} + \frac{dV}{d\phi} + \xi\mathcal{R}\phi] \tag{18}$$

for the Higgs field, where

$$\begin{aligned}\mathcal{R} = \frac{1}{\frac{1}{2} + (6\xi-1)\xi\phi^2}((1-6\xi)(\phi')^2(1-\frac{2m}{r}) + 4V(\phi) + \frac{\phi^2(1+\omega)^2}{2r^2} \\ -6\xi\phi\frac{dV(\phi)}{d\phi} - 3\xi\phi^2\frac{(1+\omega)^2}{r^2})\end{aligned} \tag{19}$$

is the spacetime curvature. The (rr) and (tt) Einstein equation are

$$\begin{aligned}(1-2\xi\phi^2)m' = (1-\frac{2m}{r})(\frac{(\phi')^2r^2}{2} + (\omega')^2 - 2\xi\phi\phi' r^2\frac{T'}{T} - 2\xi r^2(\phi')^2) + Vr^2 + \\ \frac{\phi^2(1+\omega)^2}{4} + \frac{(1-\omega^2)^2}{2r^2} - 2\xi r^2(\phi\frac{dV}{d\phi} + \frac{\phi^2(1+\omega)^2}{2r^2} + \xi\mathcal{R}\phi^2),\end{aligned} \tag{20}$$

$$\begin{aligned}(\xi\phi\phi' r - (\frac{1}{2} - \xi\phi^2))2r\frac{T'}{T} = (\frac{1}{2} - \xi\phi^2)\frac{2m}{r}\frac{1}{1-\frac{2m}{r}} + 4\xi\phi\phi' r + (\omega')^2 + \frac{r^2(\phi)'^2}{2} \\ -\frac{1}{(1-\frac{2m}{r})}(Vr^2 + \frac{\phi^2(1+\omega)^2}{4} + \frac{(1-\omega^2)^2}{2r^2}).\end{aligned} \tag{21}$$

For the black-hole solutions we replace the auxiliary T' equation with an equation for δ'

$$r\delta'(-1+2\xi\phi^2) = a_1 + \frac{a_2}{1-\frac{2m}{r}} - \frac{\frac{1}{2} + \xi\phi\phi' r - \xi\phi^2}{-\frac{1}{2} + \xi\phi\phi' r + \xi\phi^2}(a_3 + \frac{a_4}{1-\frac{2m}{r}}) \tag{22}$$

where

$$a_1 = \quad (\omega')^2 + \frac{(\phi')^2 r^2}{2} - 2\xi r^2 (\phi')^2, \tag{23}$$

$$a_2 = \quad -(\frac{1}{2} - \xi\phi^2)\frac{2m}{r} + Vr^2 + \frac{\phi^2(1+\omega)^2}{4}$$
$$+\frac{(1-\omega^2)^2}{2r^2} - 2\xi r^2(\phi\frac{dV}{d\phi} + \frac{\phi^2(1+\omega)^2}{2r^2} + \xi\mathcal{R}\phi^2), \tag{24}$$

$$a_3 = \quad 4\xi\phi\phi' r + (\omega')^2 + \frac{r^2(\phi)'^2}{2}, \tag{25}$$

$$a_4 = \quad (\frac{1}{2} - \xi\phi^2)\frac{2m}{r} - (Vr^2 + \frac{\phi^2(1+\omega)^2}{4} + \frac{(1-\omega^2)^2}{2r^2}). \tag{26}$$

Following the analysis in [6], we can already predict the boundary conditions and some general features of the finite energy solutions. Since the Yang-Mills equations are unaffected by the presence of the $\xi\Phi^2\mathcal{R}$ term in (5), the analysis presented by Greene, Mathur and O'Neill [6] remains valid: $\omega = -1$ is the only acceptable value and $\omega \leq 1$ is required for finite energy solutions. If we assume that ϕ is $O(\alpha)$ in the region $r \geq 1$, we obtain:

$$r \leq O(1/\alpha) : -1 < \omega < \frac{1}{2}(1 + \sqrt{1-(\alpha r)^2})$$
$$r \geq O(1/\alpha) : \omega' < 0, \omega'' > 0 \tag{27}$$

These constraints are valid for both regular and black-hole solutions. The Higgs equation can be written in the form

$$\frac{1}{2}\frac{d}{dr}(\frac{r^2(\phi^2)'}{RT}) = \frac{1}{\frac{1}{2} + (6\xi - 1)\xi\phi^2}[\frac{1}{2}\frac{(\phi')^2 r^2}{RT} + \frac{Rr^2}{T}(\frac{\phi}{2}\frac{dV}{d\phi} + \frac{(1+\omega)^2\phi^2}{4r^2})$$
$$+2\xi\phi^2 r^2 \frac{R}{T}(2V - \frac{\phi}{2}\frac{dV}{d\phi})]. \tag{28}$$

The obvious requirement for finite energy solution is

$$\phi\frac{dV}{d\phi}(1 - 2\xi\phi^2) < 0 \tag{29}$$

which implies that ϕ is restricted to lie between the minima of the potential, $-\alpha \leq \phi \leq \alpha$. Relation (29) provides also an upper bound on the range of ξ :

$$\xi < \frac{1}{2\alpha^2} \tag{30}$$

It is worth noting, that an investigation [19] of classical stability of a scalar field in a curved spacetime with a general coupling to gravity found, that the Higgs fields in the standard model must have $\xi \leq 0$ or $\xi \geq 1/6$.

¿From the relation (5) we can see that an effective gravitational constant is given by

$$G_{eff} = \frac{G}{1 - 2\xi\phi^2}. \tag{31}$$

Thus the condition (30) implies positivity of the effective gravitational constant. It is not possible to obtain an explicit lower bound for ξ for a given value of α. The field equations imply that $\pm\alpha$ are the only allowed values of ϕ as $r \to \infty$. We focus here on solutions with $\phi(\infty) = \alpha$ without loss of generality. The vacuum values $\omega(\infty) = -1$ and $\phi(\infty) = \alpha$ are shared both by black-holes and regular solutions. The analysis of the field equations as $r \to \infty$ gives

$$\begin{aligned}
m(r) \sim \quad & M + \frac{1}{1-2\xi\alpha^2}(2a\sqrt{2}\xi\alpha^2\beta c r^2 e^{-\sqrt{2}\alpha\beta c r} \\
& -\frac{\alpha\beta}{2\sqrt{2}c}((c^2+1)(1-4\xi) + \frac{48\xi^3\alpha^2}{\frac{1}{2}+(6\xi-1)\xi\alpha^2}))a^2r^2e^{-2\sqrt{2}\alpha\beta c r} \quad (32) \\
lnT(r) \sim \quad & ln(T_0) + \frac{M}{r}, \quad (33) \\
\delta(r) \sim \quad & -\delta_0 - 2\sqrt{2}\frac{\xi\alpha^2\beta c}{1-2\xi\alpha^2} a r e^{-\sqrt{2}\alpha\beta c r} \\
& +\frac{\alpha\beta}{2\sqrt{2}c}((c^2+1)(1-4\xi) + \frac{48\xi^3\alpha^2}{\frac{1}{2}+(6\xi-1)\xi\alpha^2})\frac{a^2 r e^{-2\sqrt{2}\alpha\beta c r}}{1-2\xi\alpha^2}, \quad (34) \\
\omega(r) \sim \quad & -1 + b e^{-\frac{\alpha r}{2}}, \quad (35) \\
\phi(r) \sim \quad & \alpha + a e^{-\sqrt{2}\alpha\beta c r}, \quad (36)
\end{aligned}$$

where $c = \sqrt{\frac{1-2\xi\alpha^2}{1+2\xi\alpha^2(6\xi-1)}}$; M, b, a are constants; $b > 0, a < 0$.

Relation (32) implies an asymptotic violation of the weak energy condition (WEC) for negative values of ξ, since $m'(r) < 0$. There exist also other classical field theories that violate the WEC. Examples are theories containing $\mathcal{R} + \mathcal{R}^2$ terms in the action[20] , an antisymmetric 3-form axion field coupled to scalar fields [21], the Brans-Dicke scalar-tensor theory [22], and Einstein-dilaton theory with curvature-squared terms of Gauss-Bonnet type [23].

Since $\mu(\xi) = \alpha\beta\sqrt{\frac{1-2\xi\alpha^2}{1+2\xi\alpha^2(6\xi-1)}}$ corresponds to the mass of the Higgs field at infinity, the following relation holds

$$\mu(\xi) \leq \mu(0) = \alpha\beta. \quad (37)$$

Thus any nonminimal coupling decreases the asymptotic value of the Higgs field mass. To get further insight into the meaning of a large value of ξ it is worthwhile to consider the rescaling $r \to r/\sqrt{(-\xi)}$, $\phi \to \phi/\sqrt{(-\xi)}$. For $\xi \to -\infty$ we find that $\alpha \to 0$ necessarily. Thus, for a large negative ξ, we expect a decrease of the maximal allowed value of the parameter α. Furthermore, there is an effective decoupling of the Yang-Mills and gravitational fields and the effective coupling of the Higgs field to matter becomes of gravitational strength. In the limit of infinite negative ξ, the following field equations are obtained

$$(\frac{1}{2}+\phi^2)2r\frac{R'}{R} = (\frac{1}{2}+\phi^2)(1-R^2) + 2\phi\phi' r^2\frac{T'}{T}, \quad (38)$$

$$(\phi\phi' r + \frac{1}{2}+\phi^2)2r\frac{T'}{T} = (\frac{1}{2}+\phi^2)(1-R^2) + 4\phi\phi' r, \quad (39)$$

$$\omega'' = \omega'(\frac{R'}{R} + \frac{T'}{T}) + \frac{\omega(\omega^2-1)R^2}{r^2} + \frac{\phi^2(1+\omega)^2}{4}R^2, \tag{40}$$

$$\phi'' = \phi'(\frac{R'}{R} + \frac{T'}{T} - \frac{2}{r}) - \frac{(\phi')^2}{\phi}. \tag{41}$$

By using a usual power series expansion near the origin or the event horizon it can be proven that there are no initial conditions consistent with the requirement of energy finiteness. However a simpler proof is to observe that equation (41) implies the relation $(\phi^2)' = const.\frac{RT}{r^2}$, which is also consistent with the general equation (28). There are no nonsingular solution of this equation consistent with the requirement of metric regularity at the origin or with a regular event horizon. Thus we conclude that nontrivial solutions are absent in the case of an infinite negative ξ. In practice, it becomes increasingly difficult to solve the field equations for large negative values of ξ, with a fast convergence to the asymptotic values $\omega(\infty)$, $\phi(\infty)$.

A particularly interesting case of the general theory is obtained for $G \to \infty$, i.e. in the absence of the Einstein term in the action (5), corresponding to the spontaneous symmetry breaking theory of gravity, with the standard Higgs field as the origin of the Plank mass (for the remainder of this paragraph we do not consider the rescaling (15)).

There has recently been an increased interest in induced gravity in the standard model, that might help to solve some problems of particle physics and cosmology. Typical problems are the necessity of the Higgs mass to be order of the theory cut-off [24], the missing mass problem, Mach's principle [25], and the inflationary scenario [26]. The existence of magnetic monopole solutions in an induced gravity YMH theory has also been discussed [27]. Unfortunately, it can be proven that, in the absence of the Einstein term, only the trivial case $\phi = \pm\alpha$ is consistent with the requirement of energy finiteness. When we consider the equation (18) and use the trace of Einstein equations to eliminate the term $\xi\phi\mathcal{R}$ we obtain the general equation

$$\frac{1}{2}\nabla_\mu\nabla^\mu\phi^2 = \frac{1}{1-6\xi}(\frac{\mathcal{R}}{2G} + \phi\frac{dV}{d\phi} - 4V). \tag{42}$$

For our ansatz we obtain the relation

$$\frac{d}{dr}(\frac{r^2(\phi^2)'}{RT}) = \frac{Rr^2}{T}\frac{1}{1-6\xi}(\frac{\mathcal{R}}{2G} + \phi\frac{dV}{d\phi} - 4V). \tag{43}$$

which should be satisfied for all r. Since clearly $4V - \phi\frac{dV}{d\phi} > 0$ for the considered potential, it follows that in the absence of an Einstein term in the original action only $\phi = \pm\alpha$ is consistent with the assumption of finite energy, both for regular and black-hole solutions. However, for $\phi = \pm\alpha$ we obtain the Bartnik-McKinnon solutions and their black-hole generalizations; there are no spherically symmetric gravitating sphaleron or sphaleron black-hole solutions. One can conjecture that similar to the boson star case, it may be possible to obtain nontrivial solutions by considering a time dependence of the matter field.

3 NUMERICAL SOLUTION

Nontrivial solutions are not known in closed form, and so a numerical method of solution is necessary.

3.1 REGULAR SOLUTIONS

For regular solutions, finite T_{tt} and regularity of the metric at $r = 0$ give two possible sets of initial conditions

$$2m(r) = O(r^3) \tag{44}$$

$$lnT(r) = O(r^2) \tag{45}$$

$$\begin{pmatrix} \omega(r) \\ \phi(r) \end{pmatrix} = \begin{pmatrix} -1 + O(r^2) \\ \phi_0 + O(r^2) \end{pmatrix}, \tag{46}$$

or

$$\begin{pmatrix} \omega(r) \\ \phi(r) \end{pmatrix} = \begin{pmatrix} 1 + O(r^2) \\ O(r) \end{pmatrix}. \tag{47}$$

The general properties of the solutions are the same as for the minimally-coupled EYMH theory. Solutions are again characterized by $w(r)$ oscillations in the region $r > 1$ and classified by the node number k which may be even or odd. The formal power series describing the above boundary conditions at $r = 0$ is

$$2m(r) = ar^3 + O(r^5), \tag{48}$$

$$lnT(r) = \frac{1}{2} fr^2 + O(r^4), \tag{49}$$

$$\omega = -1 + br^2 + O(r^4), \tag{50}$$

$$\phi = \phi_0 + er^2 + O(r^4), \tag{51}$$

with

$$a = \frac{4b^2 + \frac{2}{3}V_0}{1 - 2\xi\phi_0^2} - \frac{4}{3}\frac{\xi\phi_0}{1 - 2\xi\phi_0^2}\frac{(\frac{1}{2} - \xi\phi_0^2)V_0' + 4V_0\phi_0\xi}{\frac{1}{2} + (6\xi - 1)\xi\phi_0^2}, \tag{52}$$

$$f = -\frac{1}{2}\frac{4b^2 + \frac{2}{3}V_0}{1 - 2\xi\phi_0} + \frac{2}{3}\frac{\xi\phi_0}{1 - 2\xi\phi_0^2}\frac{(\frac{1}{2} - \xi\phi_0^2)V_0' + 4V_0\phi_0\xi}{\frac{1}{2} + (6\xi - 1)\xi\phi_0^2}$$
$$+ \frac{8\xi\phi_0 e - V_0 + 2b^2}{2\xi\phi_0^2 - 1}, \tag{53}$$

$$e = \frac{1}{6}V_0' + \frac{\xi\phi_0}{\frac{1}{2} + (6\xi - 1)\xi\phi_0^2}(4V_0 - 6\xi\phi_0 V_0'), \tag{54}$$

for even-k solutions (V_0, V_0' are the potential and its derivative with respect to ϕ at $\phi = \phi_0$) and

$$2m(r) = (4b^2 + \frac{2}{3}V_0 + e^2 - 4\xi e^2)r^3 + O(r^5), \tag{55}$$

$$lnT(r) = -(2b^2 - \frac{1}{3}V_0 + 2\xi e^2)r^2 + O(r^4), \tag{56}$$
$$\omega = 1 - br^2 + O(r^4), \tag{57}$$
$$\phi = er + O(r^3), \tag{58}$$

for odd-k solutions. The shooting parameters are (ϕ_0, b) and (e, b) respectively. Using a standard ordinary differential equation solver, we evaluate the initial conditions at $r = 10^{-3}$ for global tolerance 10^{-12}, adjusting for fixed shooting parameters and integrating towards $r \to \infty$. The difficulty of the two-dimensional shooting problem in the presence of two free parameters is increased by the presence of a nonminimal coupling between the Higgs field and gravity which leads to a slow convergence of the mass function $m(r)$.

We limit the discussion to results containing only one or two nodes. The results obtained for the $k = 1$ and $k = 2$ solutions retain the general characteristics of the minimally coupled case. In order to define the terminology, which is somewhat confusing, we review here the $\xi = 0$ case ([5, 6, 18]).

In the $k = 1$ case there are two possible solutions that are called the quasi k=0 solution and the proper $k = 1$ solution. The different character of the two branches becomes apparent in the limit $\alpha \to 0$. Remembering that $\alpha = v\sqrt{G}$, there are two physically inequivalent ways for α to approach 0. The first is for the Newton constant to vanish, the second for the Higgs vacuum expectation value to vanish. In the first case we have weakly coupled gravity and the solution approaches the standard model sphaleron. The quasi-k =0 branch of solutions has this limit for $\alpha \to 0$. For $\alpha \to 0$ the node that is present actually moves to infinity in this branch, therefore the name quasi-k =0. The proper $k = 1$ branch approaches the first Bartnik-McKinnon solution when $\alpha \to 0$. It corresponds to taking $v \to 0$, but always having gravity present. For small enough α, depending on β, both solutions are present and different. For finite α the behaviour is dependent on the value of β. For β larger than a critical value $\beta_{crit} \approx 0.12$ there is a first maximum value of α, where the proper $k = 1$ branch disappears. For a larger value of α also the quasi-$k = 0$ solution disappears. In the case β smaller than β_{crit} the maximum value of α is the same for both branches. Here the solutions merge; the shooting parameters approach each other. The situation is clarified in $figure$ 1.

For the case of two nodes the situation is similar. There are again two branches, called the quasi$-k = 1$ solution and the proper $k = 2$ solution. The quasi-$k = 1$ solution approaches the first Bartnik-McKinnon solution in the limit $\alpha \to 0$, one of the nodes moving to infinity. The proper $k = 2$ solution approaches the second Bartnik-McKinnon solution. The behaviour as a function of α and β is qualitatively similar to the one-node case ($figure$ 1).

To compare numerically the results with those found in [6] we focused on solutions with $\beta^2 = 1/8$ (although similar results have been obtained for other choices of β) and with $k = 1$ and $k = 2$ only. The results of the numerical integration for α=0.1, β^2=1/8 and a range of ξ are presented in $figure$ 2.

In this figure ξ is relatively small (i.e. $\xi \ll \frac{1}{2\alpha^2}$) . As a consequence the results of [6] remain approximately valid. The correction to the shooting pa-

rameters is very small. A general feature of the solutions is the small influence of the term $\xi\Phi^2\mathcal{R}$ on the value of the ADM mass. A negative value of ξ seems to decrease the asymptotic value of the $m(r)$ function while a positive ξ determines a higher ADM mass with respect to the minimally coupled case. The effect is particularly small for the quasi-$k = 0$ solution. This is understandable as this solution corresponds to the flat space sphaleron, for which gravity is a small effect. For the considered range of ξ, a nonminimal coupling term has a small effect on the shape of the $w(r)$ function. However, for suitable values of ξ we have noticed a strong influence of this term on the behavior of the Higgs field in the intermediate region. Also, for k=2 solutions, the initial value of the Higgs field ϕ_0 is strongly dependent on the value of ξ, with $\phi_0(\xi \neq 0) < \phi_0(\xi = 0)$ ($figure$ 2 c, d) . As we expected from (32), for negative ξ we obtain a violation of the WEC beyond a certain limit of the radial coordinate, corresponding to a peak of the mass function $m(r)$. The height of the peak is proportional to the absolute value of ξ.

In $figure$3 we study the solutions as a function of α. Significant changes occur for $\xi \rightarrow \frac{1}{2\alpha^2}$ and for large negative values of ξ. The parameter range obtained in [6] for the two sheets of solutions does not remain valid; it is necessary to establish a different value of α_{max} for every choice of ξ. A general feature for a nonminimal coupling is the decrease of the maximal allowed value of the parameter α . For example for the proper $k = 1$ branch, Greene, Mathur and O'Neill [6] have found $0 < \alpha < 0.599$; for $\xi = 0.1$ we have obtained $0 < \alpha < 0.48$, while for $\xi = -2$ the limiting value of the parameter α is 0.51 ($figure$ 3a, b). The quasi-k=0 branch with $\xi = 0$ has $0 < \alpha < 0.619$; for $\xi = -1$ we have found $\alpha_{max} = 0.616$ while for $\xi = 1/6$ we have $0 < \alpha < 0.525$ ($figure$ 3c, d). A minimally coupled even k configuration has $0 < \alpha < 0.120$ (proper $k = 2$ branch) or $0 < \alpha < 0.122$ (quasi-$k = 1$ branch); in these cases, for $\xi = -1$ or $\xi = 1/6$ we did not notice a significant deviation of the α_{max} value ($figure$ 3 e-h). Different limiting values occur for the shooting parameters b, ϕ_0 and e also.

3.2 BLACK-HOLE SOLUTIONS

Similar results can be obtained for numerical black-hole solutions. We use the following expansion near the event horizon:

$$m(r) = \frac{r_h}{2} + m'(r_h)(r - r_h), \tag{59}$$

$$\delta(r) = 0 + \delta'(r_h)(r - r_h), \tag{60}$$

$$\omega(r) = \omega(r_h) + \omega'(r_h)(r - r_h), \tag{61}$$

$$\phi(r) = \phi(r_h) + \phi'(r_h)(r - r_h), \tag{62}$$

with

$$m'(r_h) = \frac{1}{2} + \frac{1}{2}\frac{a_2(r_h) - a_4(r_h)}{1 - 2\xi\phi^2(r_h)}, \tag{63}$$

$$\omega'(r_h) = -\frac{r_h(1-2\xi\phi^2(r_h))}{a_2(r_h) - a_4(r_h)}(\frac{\omega(r_h)(\omega^2(r_h)-1)}{r_h^2}$$
$$+\frac{\phi^2(r_h)(1+\omega(r_h))}{4}), \tag{64}$$

$$\phi'(r_h) = \frac{1-2\xi\phi^2(r_h)}{2\xi\phi(r_h)r_h}\frac{a_2(r_h)+a_4(r_h)}{a_2(r_h)-a_4(r_h)}, \tag{65}$$

$$\delta'(r_h) = \frac{1}{r_h(1-2\xi\phi^2(r_h))}(\frac{f'(r_h)r_h}{1-2m'(r_h)}\frac{1}{-\frac{1}{2}+\xi\phi^2(r_h)+\xi\phi(r_h)\phi'(r_h)r_h}$$
$$+\frac{(a_1(r_h)+a_3(r_h))(\frac{1}{2}-\xi\phi^2(r_h)) - \xi\phi(r_h)\phi'(r_h)r_h(a_1(r_h)-a_3(r_h))}{-\frac{1}{2}+\xi\phi^2(r_h)+\xi\phi(r_h)\phi'(r_h)r_h}) \tag{66}$$

where

$$f(r) = a_2(r) + a_4(r))(\frac{1}{2} - \xi\phi^2) - \xi\phi\phi' r(a_2(r) - a_4(r)). \tag{67}$$

The new shooting parameters are $\omega(r_h)$ and $\phi(r_h)$ (we have studied the case $r_h = 1$ only).

The non-minimal gravitational coupling allows for a not necessarily positive field energy. Therefore one loses one of the earlier tools for proving the no hair theorems, which already failed for the minimally coupled EYMH system. The bypassing of the usual no-hair theorems in the considered system can be proven by using the method of [7] for the $\xi = 0$ case.

Starting from the solutions (59-62) we integrated the system (20, 22, 17, 18) towards $r \to \infty$ using an automatic step procedure and accuracy 10^{-12}. The integration stops when the flat spacetime asymptotic limit (32, 33, 34, 35) is reached.

The behaviour of the black-hole solutions as a function of ξ and α is similar to the regular solutions. Two solution branches appear for each k corresponding to two different values of the shooting parameters $\omega(r_h)$ and $\phi(r_h)$. As $\alpha \to 0$, the proper $k = 1, 2$ branches approaches the corresponding Einstein-Yang-Mills black-hole solutions ([6, 7]).

In the same limit, the quasi-$k = 0$ branch is distinguished by its Schwarzschild solution limit ($\omega = 1$, $\phi = 0$) and the last node of the quasi-$k = 0$ and quasi-$k = 1$ branches is again pushed out to infinity. The corresponding limit of the quasi-$k = 1$ branch is the n=1 Einstein-Yang-Mills black-hole solution.

Again, every branch the solutions exist only for a finite range of the parameter $0 \leq \alpha \leq \alpha_{max}(\beta, k)$ with different values of α_{max} for every solution branch.

The results for k =1, 2, β^2=1/8 and various values of the parameter ξ are presented in $figure$ 4. As we expected, for a nonzero ξ it is necesary to establish new limiting values of the values of the normalised vacuum expectation values

α. For example, a minimally coupled solution has necessarily $0 < \alpha < 0.331$ (proper $k = 1$ branch), $0 < \alpha < 0.356$ (quasi-$k = 0$ branch), $0 < \alpha < 0.0475$ (proper $k = 2$ branch) and $0 < \alpha < 0.0486$ for the quasi-$k = 2$ branch.

We have found $0 < \alpha < 0.325$ ($\xi = -1$) and $0 < \alpha < 0.352$ for $\xi = 1/6$ (proper $k = 1$ branch); some results for the quasi-$k = 0$ branch are: $0 < \alpha < 0.356$ ($\xi = -0.1$), $0 < \alpha < 0.352$ ($\xi = 1/6$). For the proper $k = 2$ branch we have found $0 < \alpha < 0.0382$ ($\xi = 60$), $0 < \alpha < 0.336$ ($\xi = -60$), while the quasi-$k = 0$ branch with $\xi = 10$ has the limiting value $\alpha < 0.044$; for a negative coupling constant $\xi = -5$ we have found $0 < \alpha < 0.047$ (see also $figure$ 5). Different ranges for the shooting parameters $\omega(r_h)$ and $\phi(r_h)$ are to be imposed.

Similar to the case of regular solutions, we notice the occurence of negative energy densities. Anyway, an unexpected feature is the violation of the WEC even in the vicinity of the event horizon (for positive values of ξ and quasi$-k = 0$ branch), which is supposed to destabilize the black-hole and to lead to a traversable wormhole [28].

Another interesting problem is the effect of the nonminimal coupling on the properties of a black-hole. Not suprisingly, for the quasi$-k = 0$ branch we have noticed a violation of the generic relation [28]

$$T_H = \frac{1}{4\pi r_H}\, e^{-\delta(r_H)}\, (1 - 2m'(r_h)) \leq \frac{1}{4\pi r_h} = T_H^{vac}. \tag{68}$$

(we use units $k_B = \hbar$=1) where T_H^{vac} is the Hawking temperature of a Schwarzchild black-hole with the same area.

We have found that generally a positive ξ will increase the value of the Hawking temperature (the only disturbing exception is the quasi $k = 1$ case). However, following [29] the validity of the generic relation $S = \frac{A}{4}$ (where A is the event horizon area) can easily be proven for the Lagrangian density (5).

4 FURTHER DISCUSSION

Further insight into the meaning of the nonminimal coupling in EYMH theory can be obtained by using the conformal rescaling of the action (5):

$$\overline{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 - 2\xi G\phi^2. \tag{69}$$

The use of this conformal transformation together with a redefinition of the scalar field for the case of nonminimal coupling has a long history; ref. [15], for instance, presents a large set of references on this subject. The usual condition $\Omega^2 > 0$ has a clear physical meaning since it is satisfied by finite energy solutions only. The pairs of variables (metric $g_{\mu\nu}$, scalar ϕ, $SU(2)$ field $F_{\mu\nu}$) defined originally constitute what is called a Jordan frame. Consider now the transformation

$$\psi = \int d\phi F(\phi),$$
$$F^2(\phi) = \frac{1 - 2\xi G\phi^2(1 - 6\xi)}{(1 - 2\xi G\phi^2)^2} \tag{70}$$

such that, in the redefined action

$$S = \int d^4x \sqrt{-\overline{g}}[\frac{\overline{\mathcal{R}}}{16\pi G} - \frac{1}{4\pi}\frac{1}{2}(\partial_\mu \psi)(\partial^\mu \psi)$$

$$-\frac{1}{4\pi}\frac{1}{2}(\frac{g\phi}{2})^2 \frac{\mid A \mid^2}{1-2\xi G\phi^2} - \frac{1}{4\pi}\overline{V}(\psi) - \frac{1}{4\pi}\frac{1}{4} \mid F \mid^2] \tag{71}$$

ψ becomes minimally coupled to $\overline{\mathcal{R}}$, with

$$\overline{V}(\psi) = \frac{V(\phi)}{\Omega^4}. \tag{72}$$

The new variables (metric $\overline{g}_{\mu\nu}$, scalar ψ, $SU(2)$ field $F_{\mu\nu}$) are said to constitute an Einstein frame. The transformation given by eqs. (69, 70) therefore maps a solution of the field equations imposed by (5) to a solution that extremizes (71). The transformation is independent of any assumption of symmetry, and in this sense is covariant; one can easily infer that the transformation is one-to-one in general. Also, the transformation preserves symmetries, which means that if $g_{\mu\nu}$ admits a Killing vector η such that $\pounds_\eta \phi = 0$, then η is also a Killing vector of $\overline{g}_{\mu\nu}$ and $\pounds_\eta \psi = 0$. There is a long debate in the literature on the problem of which of these two frames is physical (for a review see [15, 30]). For example, in ref. [31] it has been shown in a more general context that all thermodynamical variables defined in the original frame are the same as those in the Einstein frame, if spacetimes in both frames are asymptotically flat, regular and posses event horizons with non-zero temperature. We know that Ω^2 goes to some finite positive value at infinity. Since this value is not unity, the asymptotically Minkowskian metric $g_{\mu\nu}$ will be mapped into a generally non-asymptotically Minkowskian line element $\overline{g}_{\mu\nu}$. However, one needs only to redefine globally the units of length and time to obtain an Einstein-frame standard Minkowski form at infinity. Considering an expansion of the Higgs field ϕ around the minimum $\phi = v + \eta$, for large enough negative values of ξ we obtain the following first order Einstein frame expression

$$L_{YM} = -\frac{1}{4\pi}\frac{1}{4} \mid F \mid^2 + \frac{1}{4\pi}\frac{1}{4\xi G}(\frac{g}{4})^2 \mid A \mid^2, \tag{73}$$

with an effective decoupling of the YM and Higgs fields and a massive Yang-Mills theory, along the line suggested in [24].

The Weyl rescaling (69) helps us to rule out the existence of traversable wormhole solutions, since one can conclude that when we know all Einstein-frame solutions with a given symmetry we automatically know all Jordan-frame solutions with the same symmetry.

A spacetime wormhole is usually introduced as a topological handle connecting two universes or distant places in the same universe. Over the last decade following the seminal papers of Morris, Thorne and Yurtsever [32, 33], considerable interest has grown in the domain of traversable wormhole physics (for a review see [34]). We recall that a Lorentzian wormhole solution is said

to be traversable if it does not contain horizons that prevent the crossing of the throat. A remarkable result is that, assuming Einstein gravity, the WEC is violated at throat of a traversible static wormhole [32, 35].

Since we have found that the violation of the WEC is possible, it is natural to look for spherically symmetric, traversable wormhole solutions of the coupled EYMH equations. Further, it has been conjectured that a violation of the WEC in the vicinity of the event horizon is quite likely to destabilize the horizon and lead to a traversable wormhole [28]. Thus, we suppose the existence of a traversable wormhole solution in the original Jordan frame, therefore the violation of the energy condition at or near the throat of the wormhole. The case of a static spherically symmetric Lorentzian wormhole corresponds to the following choice of the metric functions in the general ansatz (8)

$$R(r) = (1 - b(r)/r)^{-1/2}, \quad T(r) = e^{-f(r)}, \tag{74}$$

where $b(r)$ is called the shape function as it describes the shape of the spatial geometry of the wormhole in an embedding diagram and $f(r)$ describes the gravitational redshift in this spacetime (with $e^{f(r)} > 0$) [32, 34]. In this case the coordinate r is constrained to run between $r_0 < r < \infty$, where r_0 is the throat radius ($b(r_0) = r_0$). By following the analysis of Sec. 2, we can again predict the general features of the possible solutions and boundary conditions. It follows that the general relations obtained as $r \to \infty$ are still valid. If we do not allow for ϕ or ϕ' to take an infinite value at the wormhole throat, a similar analysis of the Higgs field equation (28) implies that the generic conditions $\Omega^2 > 0$, $\xi < \frac{1}{2\alpha^2}$ hold also .

Using the conformal transformation (69) we convert the theory to the Einstein frame. The existence of wormhole solutions is not affected by the transformation (69) that preserves the traversability for $\Omega^2 > 0$, i.e. a positive effective gravitational constant and no event horizon in the new frame. It can easily be proven that for the rescaled action (71), the dominant energy condition holds, and thus there are no traversable wormhole solutions, i.e. no traversable wormhole solutions in the Jordan frame also.

One can wonder whether this absence of traversable wormhole solutions is a general feature of arbitrary nonminimal scalar couplings to Einstein gravity. The nonminimal coupling of the scalar field considered in this paper is a particular case of a more general theory, where the term $\frac{\mathcal{R}}{16\pi G}(1 - 2\xi G\phi^2)$ is replaced by a more general function $\frac{\mathcal{R}}{16\pi G} f(\phi)$.

By using a conformal transformation $\overline{g}_{\mu\nu} = f(\phi) g_{\mu\nu}$ and a redefinition of the scalar field ([15])

$$\psi = \int d\phi \left(\frac{f(\phi) + \frac{3}{4G}\left(\frac{df(\phi)}{d\phi}\right)^2}{f(\phi)^2} \right)^{1/2} \tag{75}$$

we can convert the action to the Einstein frame

$$S = \int d^4x \sqrt{-\overline{g}} [\frac{\overline{\mathcal{R}}}{16\pi G} - \frac{1}{4\pi}\frac{1}{2}(\partial_\mu \psi)(\partial^\mu \psi)$$

$$-\frac{1}{4\pi}\frac{1}{2}(\frac{g\phi}{2})^2\frac{\mid A\mid^2}{f(\phi)}-\frac{1}{4\pi}\overline{V}(\psi)-\frac{1}{4\pi}\frac{1}{4}\mid F\mid^2] \tag{76}$$

(with $\overline{V}(\psi)=\frac{V(\phi)}{f(\phi)^2}$).

If $f(\phi)>0$ (i.e. a positive effective Newton constant) the WEC will be satisfied in the Einstein frame. The positivity of the effective Newton constant followed in our case from the demand that the energy of the solution is finite; implying $\xi<\frac{1}{2\alpha^2}$. We have not been able to show that an equivalent condition holds in the general case. The possibility of traversable wormholes in general is therefore left open, though it seems likely they will be absent.

5 CONCLUSIONS

In this paper we have studied static, spherically symmetric classical solutions of spontaneously broken SU(2) gauge theory with a nonminimally coupled Higgs field and presented strong numerical arguments for the existence of both regular and black-hole solutions for suitable values of the coupling parameter ξ. The main properties of these solutions, such as their nodal structure and discrete mass spectrum, are generic and shared by practically all known solitons with gravitating non-abelian gauge fields.

It should be stressed that it is the non-abelian nature of the Yang-Mills field that allows the existence of nontrivial solutions since a nonminimally coupled Higgs charged hair has been ruled out for Abelian Higgs theory ([36], see also [37]).

The absence of gravitating sphalerons and sphaleron black-holes in a spontaneously broken theory of gravity has also been proven. As a new feature we have established a violation of the WEC for a certain range of ξ. The nonexistence of traversable wormhole solutions has been shown using a conformal map to convert the problem to the one with minimal coupling to gravity. For small values of the parameter ξ, the effect of the nonminimal coupling on the asymptotic features of a finite energy solution is rather benign. Although we did not address the nature of the solutions for $r<r_h$, we expect that a term $\xi\Phi^2\mathcal{R}$ can strongly influence the properties of the inner black-hole solutions.

We have not considered the question of stability of solutions in this paper. Since in the case of minimal coupling the solutions were found to be unstable, we see no reason to expect something special to happen for $\xi\neq 0$.

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Figure Captions

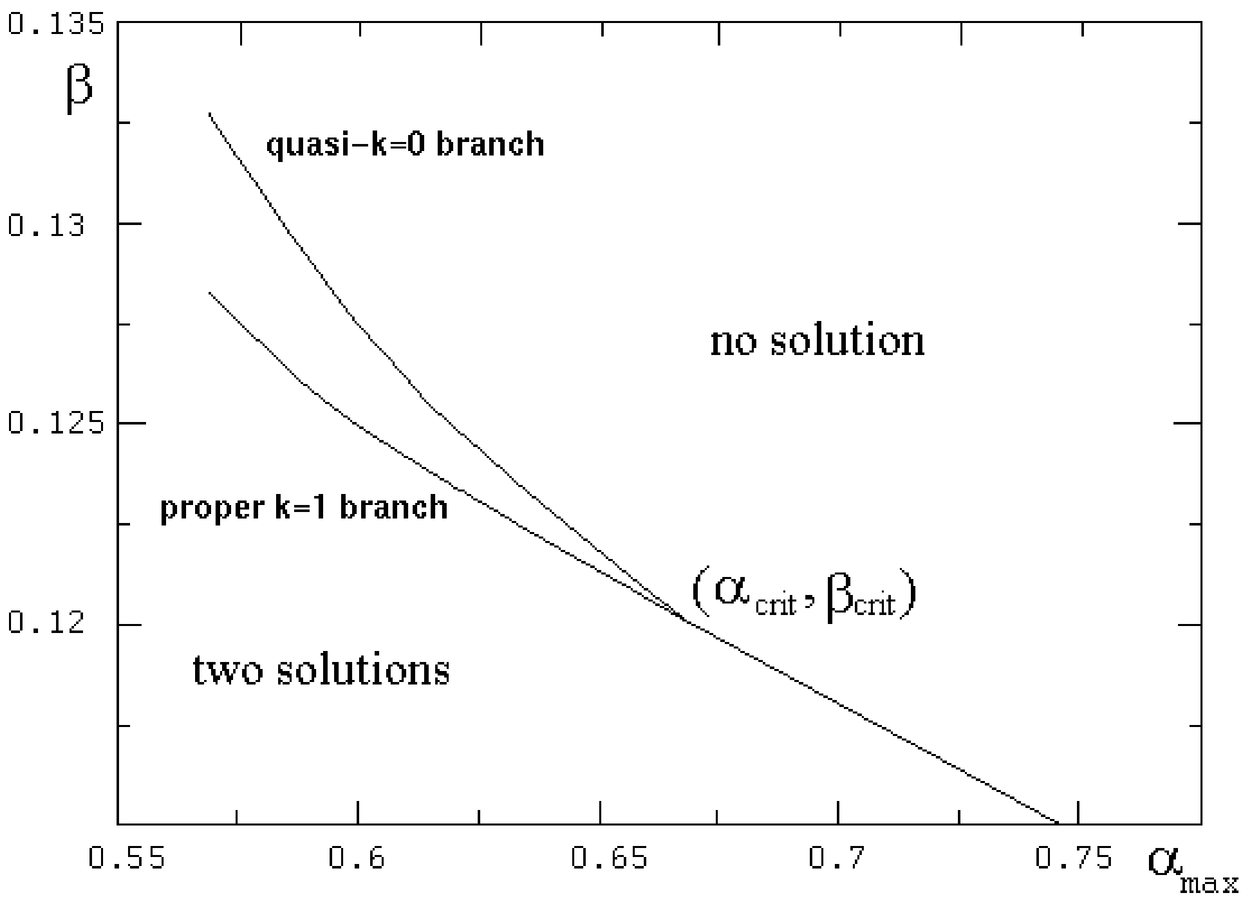
Figure 1: The parameter beta versus the maximal value of the parameter α for one node graviting sphaleron solutions; qualitative picture for the minimally coupled case.

Figure 2: One- and two-node sphaleron solutions of the nonminimally coupled EYMH theory for $\alpha = 0.1$, $\beta^2 = 1/8$ and various values of ξ.

Figure 3: One- and two-node sphaleron solutions of the nonminimally coupled EYMH theory for $\beta^2 = 1/8$ and various values of ξ. Here and in $figure$ 5, the parameter α varies between zero and the maximum allowed value α_{max}; increasing α corresponds to a decrease of the value of the radial coordinate at which the solution exponentially decays to its vacuum value.

Figure 4: One- and two-node black hole solutions of the nonminimally coupled EYMH theory for $\beta^2 = 1/8$ and various values of ξ.

Figure 5: One- and two-node black hole solutions of the nonminimally coupled EYMH theory for $\beta^2 = 1/8$ and various values of ξ.



0.135
β
0.13
0.125
0.12
quasi−k=0 branch
no solution
proper k=1 branch
(α crit , β crit)
two solutions
0.55
0.6
0.65
0.7
0.75
α max

Figure 1.

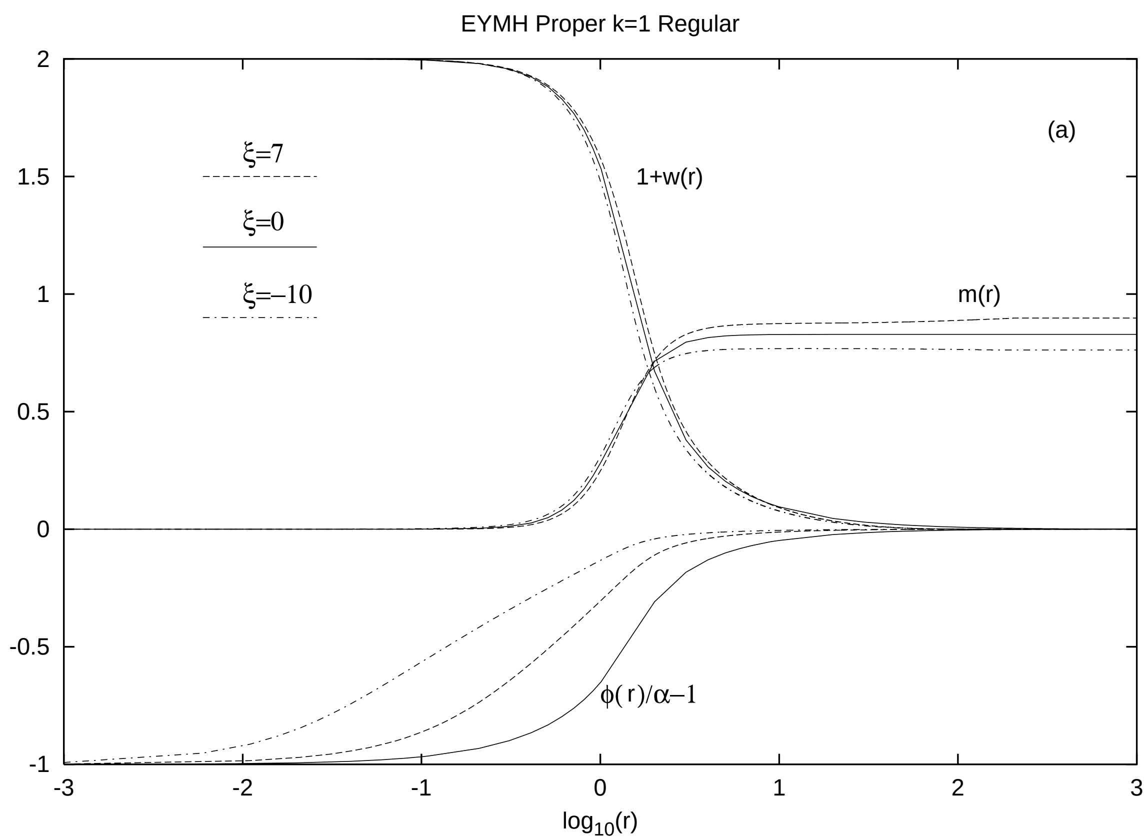


Figure 2a. Proper $k = 1$ Regular $\xi = 7$, 0, -10

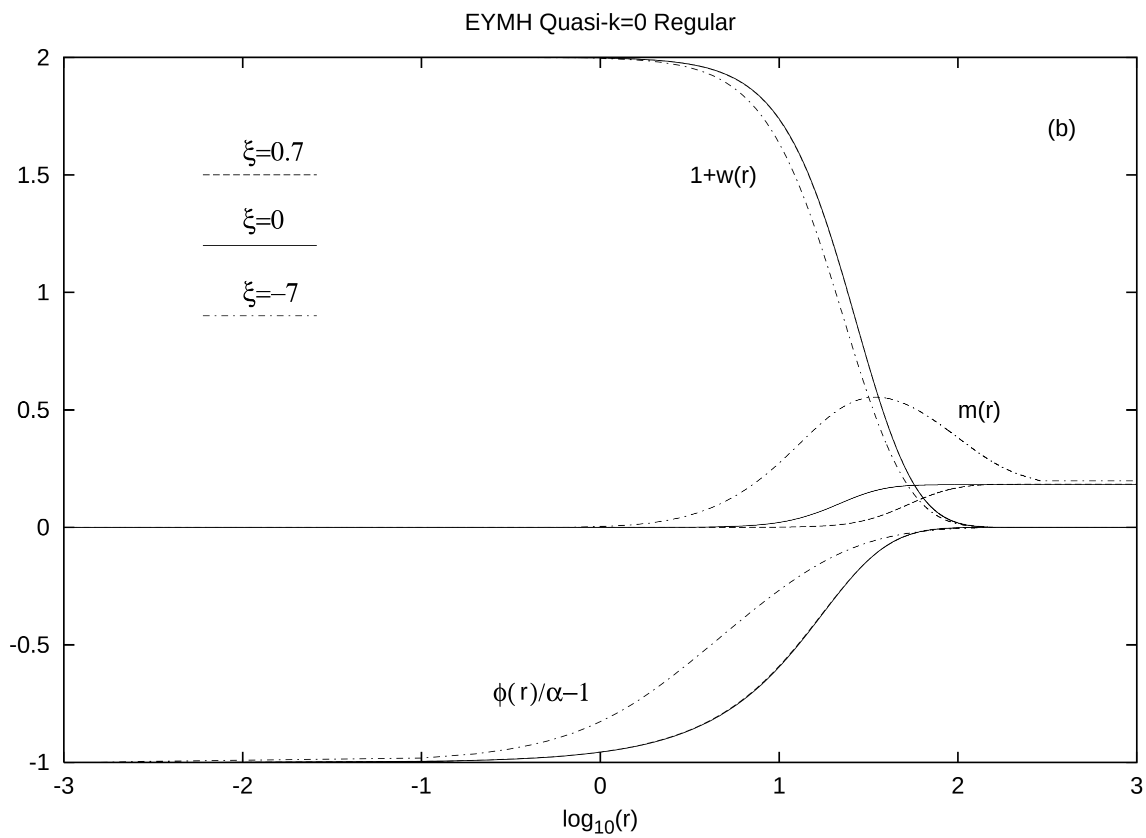


Figure 2b. Quasi$-k = 0$ Regular $\xi = 0.7$, 0, -7

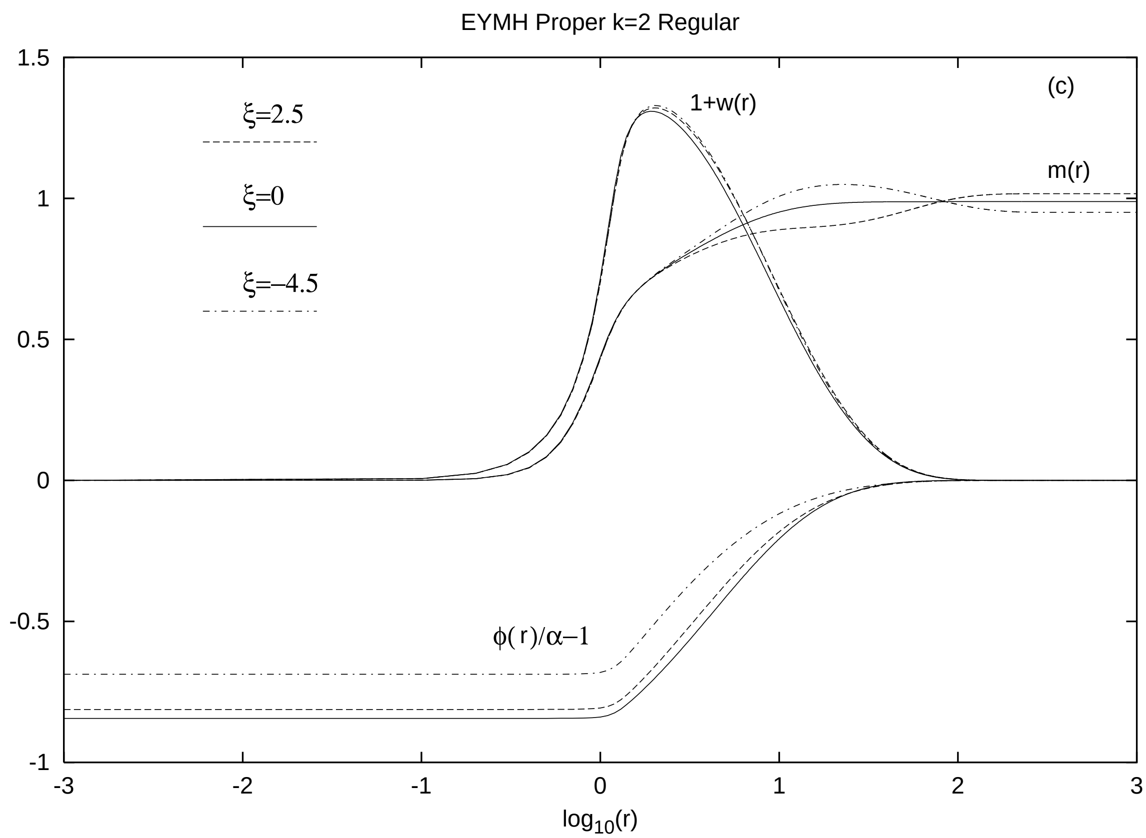


Figure 2c. Proper $k = 2$ Regular $\xi = 2.5$, 0, -4.5

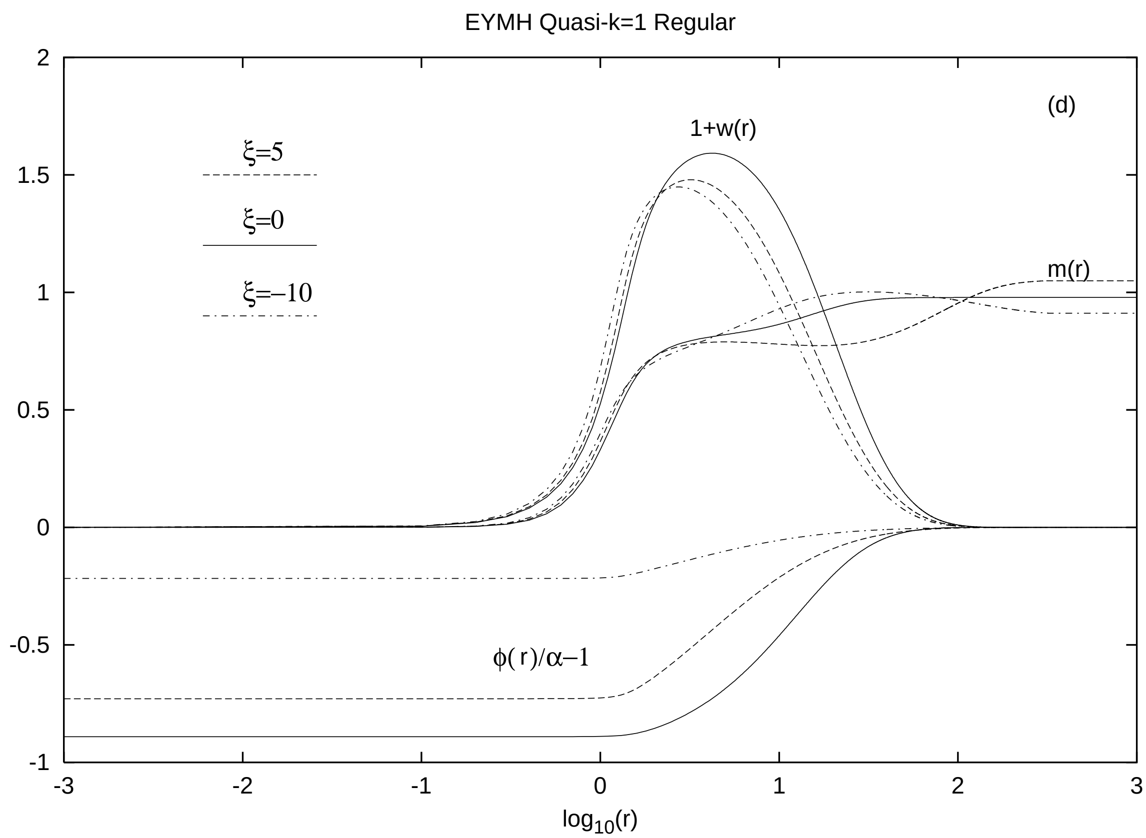


Figure 2d. Quasi$-k = 1$ Regular $\xi = 5$, 0, -10

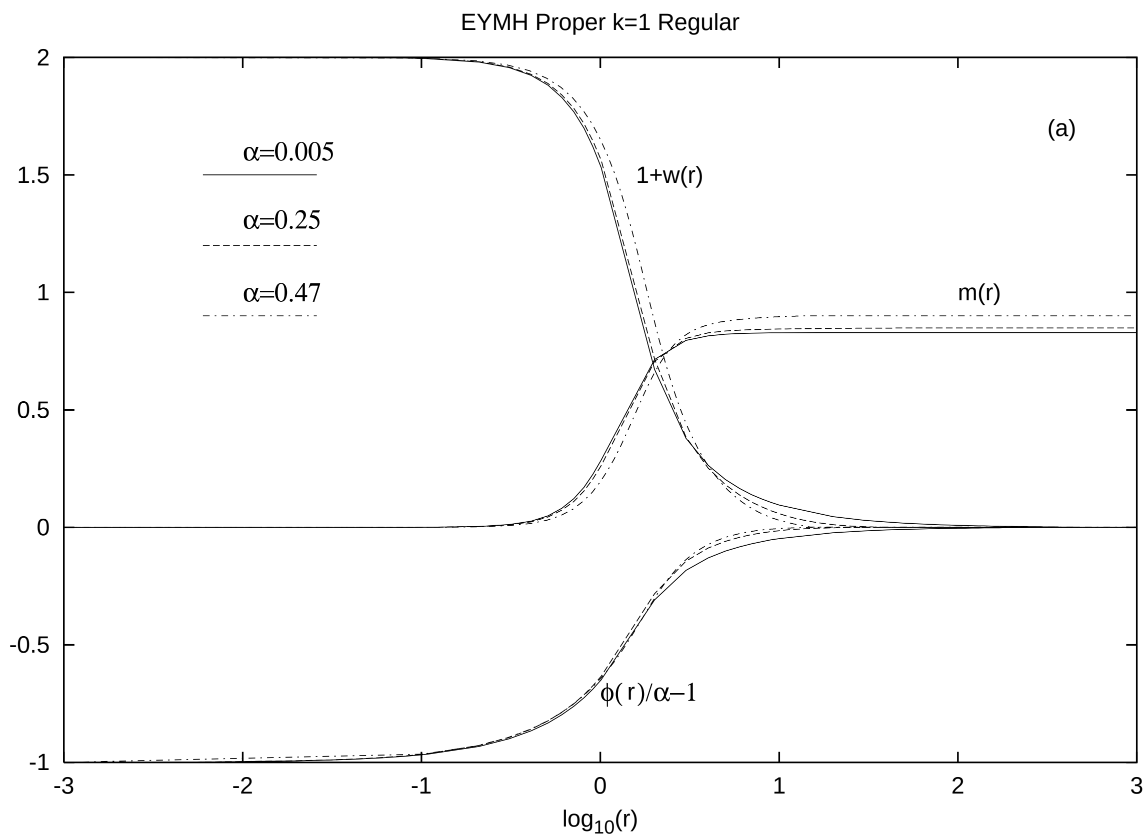


Figure 3a. Proper $k = 1$ Regular $\xi = 0.1$; $\alpha = 0.005, 0.25, 0.47$

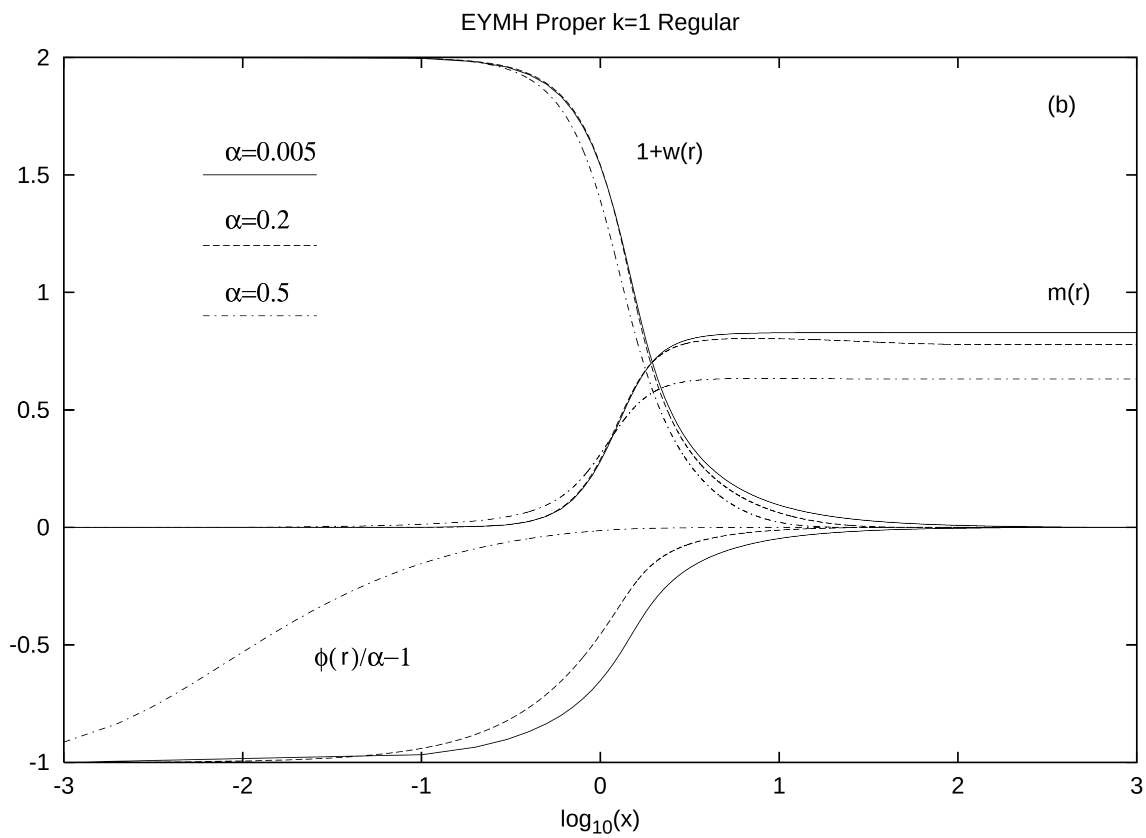


Figure 3b. Proper $k = 1$ Regular $\xi = -2$; $\alpha = 0.005, 0.2, 0.5$

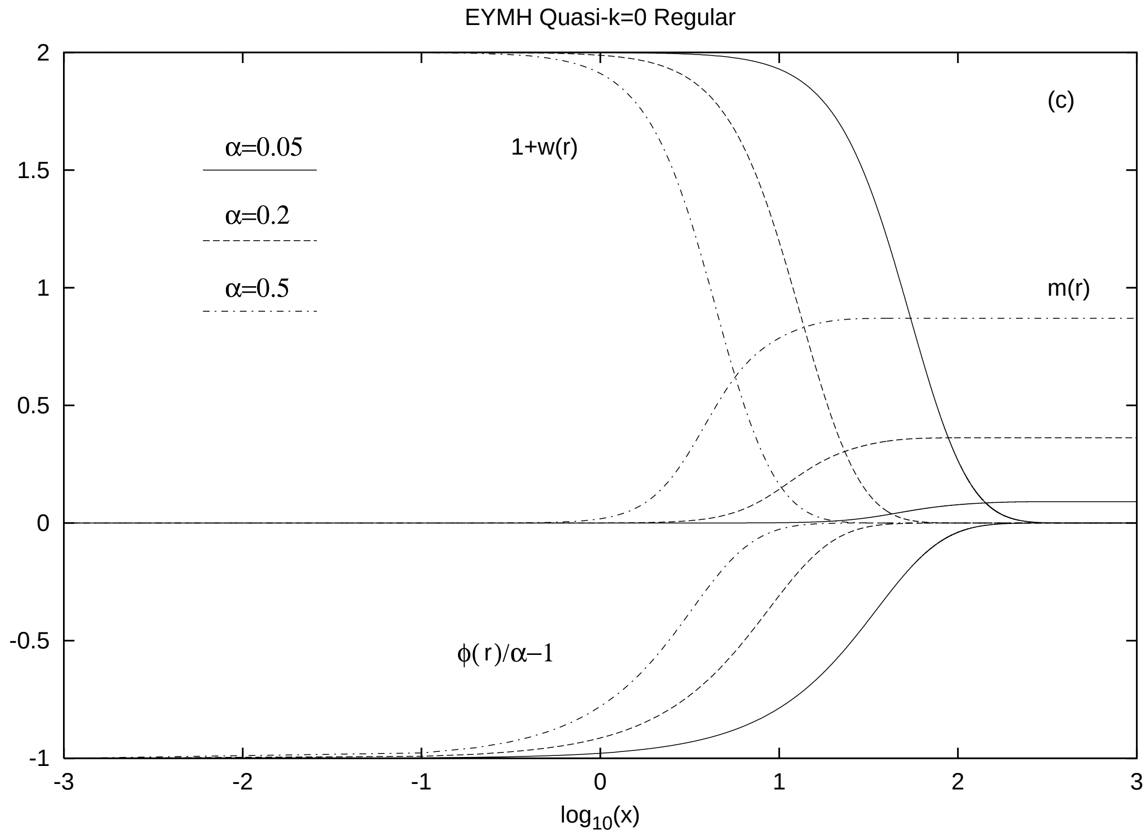


Figure 3c. Quasi$-k = 0$ Regular $\xi = 1/6$; $\alpha = 0.005$, 0.2, 0.5

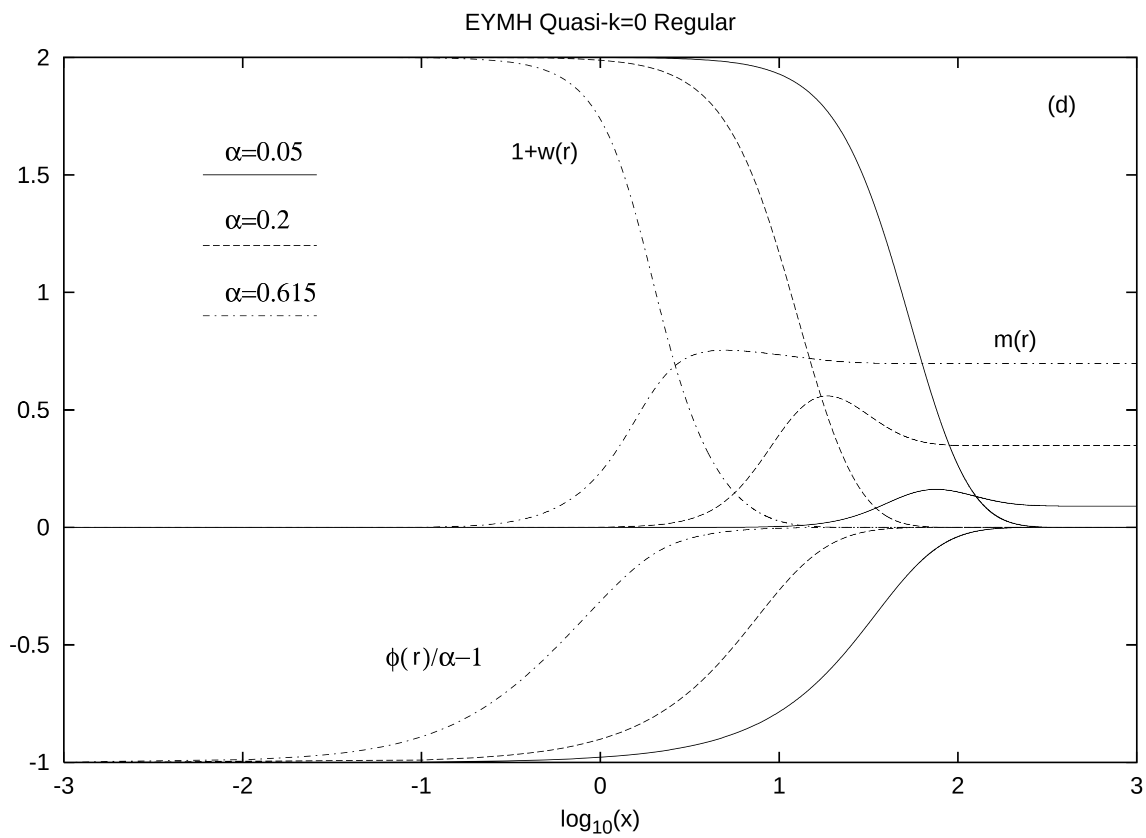


Figure 3d. Quasi$-k = 0$ Regular $\xi = -1$; $\alpha = 0.05,\, 0.2,\, 0.615$

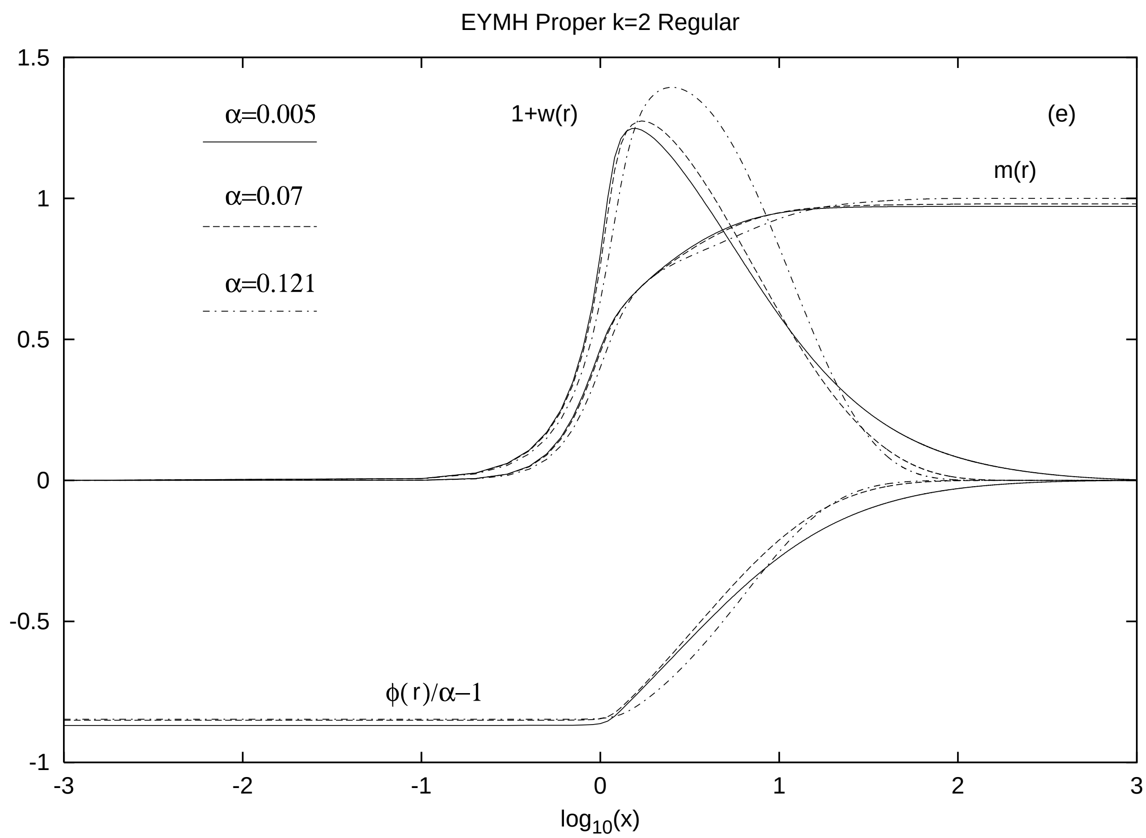


Figure 3e. Proper $k = 2$ Regular $\xi = 1/6$; $\alpha = 0.005,\, 0.07,\, 0.121$

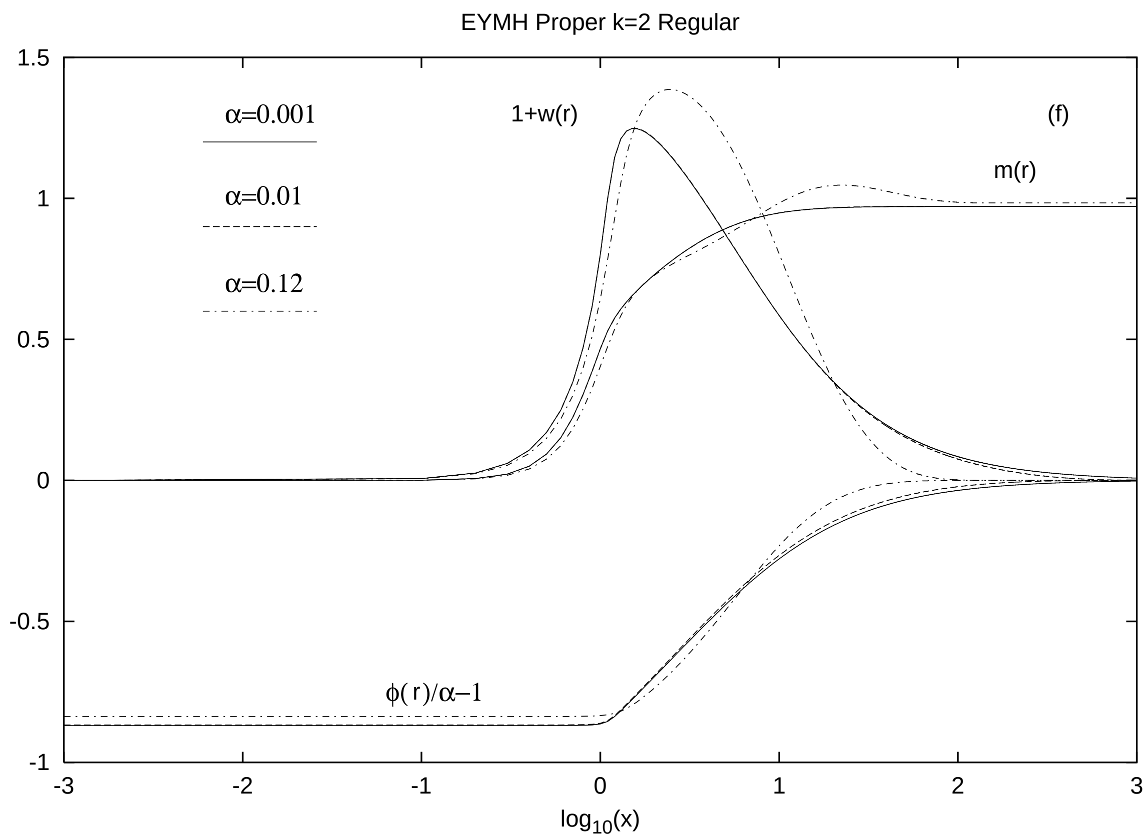


Figure 3f. Proper $k = 2$ Regular $\xi = -1$; $\alpha = 0.001,\ 0.01,\ 0.12$

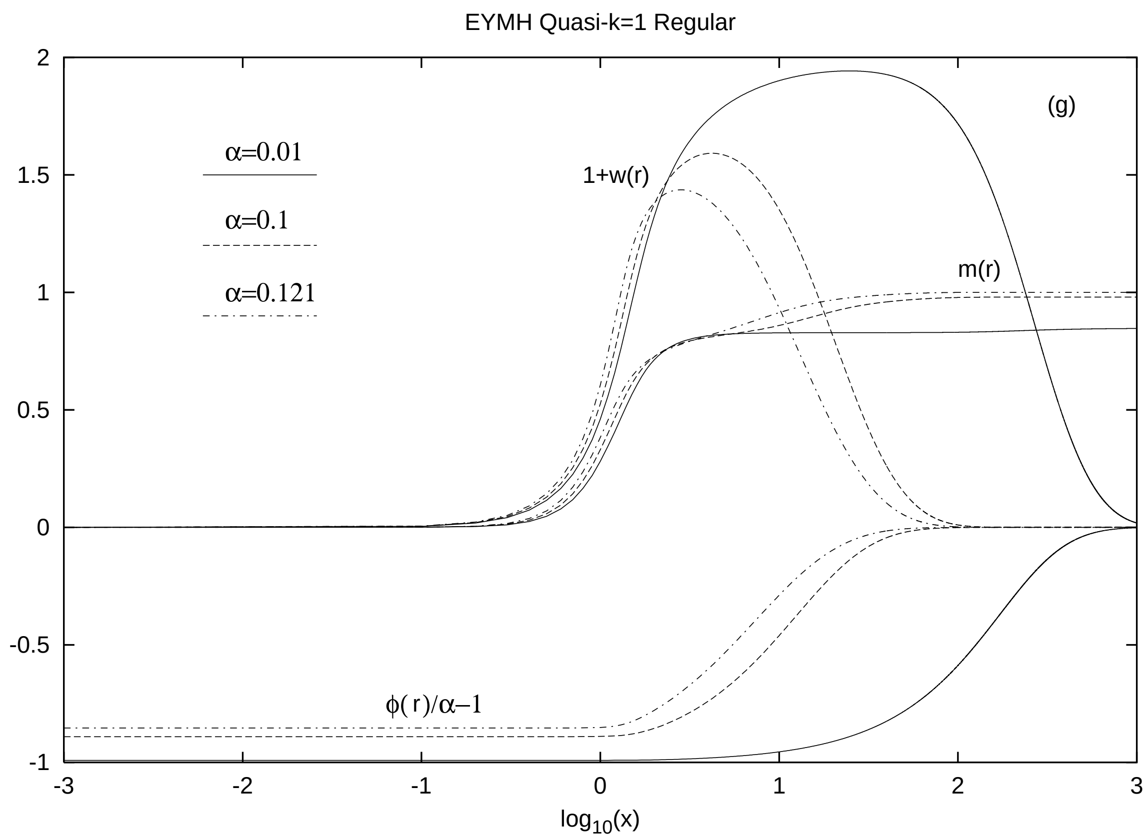


Figure 3g. Quasi$-k = 1$ Regular $\xi = 1/6$; $\alpha = 0.01$, 0.1, 0.121

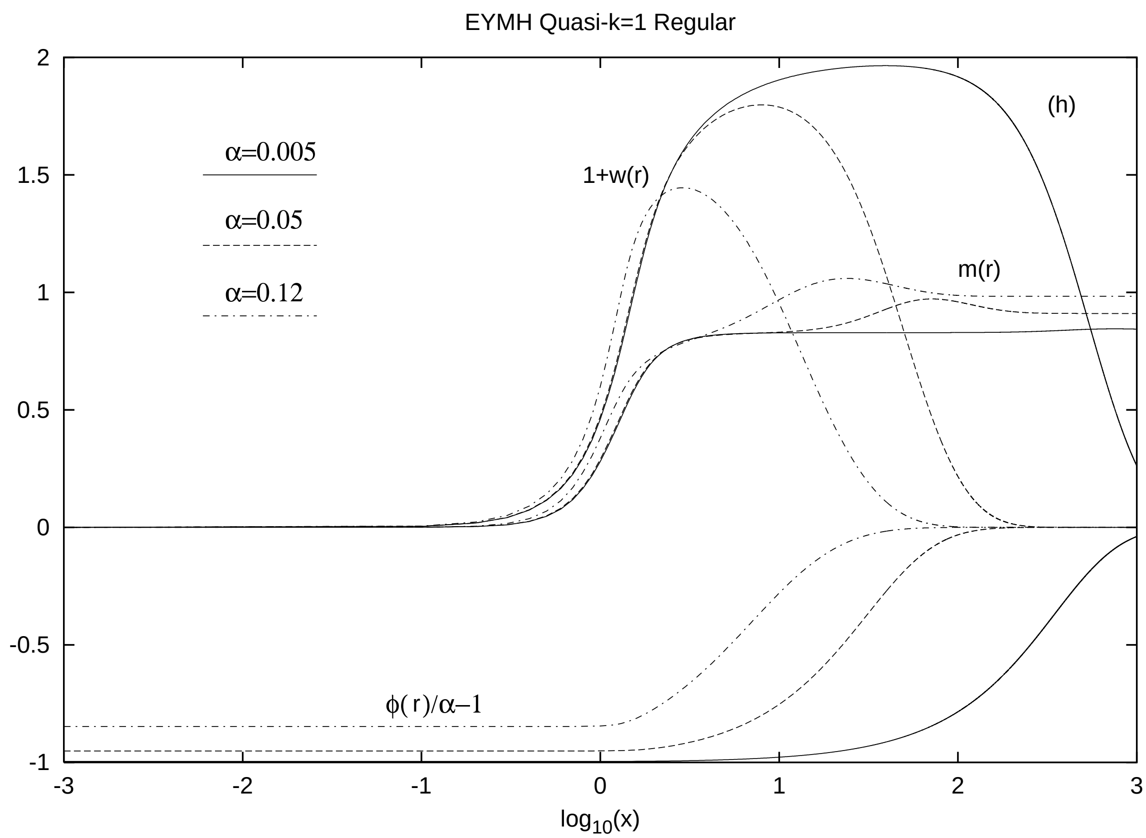


Figure 3h. Quasi$-k = 1$ Regular $\xi = -1$; $\alpha = 0.005,\, 0.05,\, 0.12$

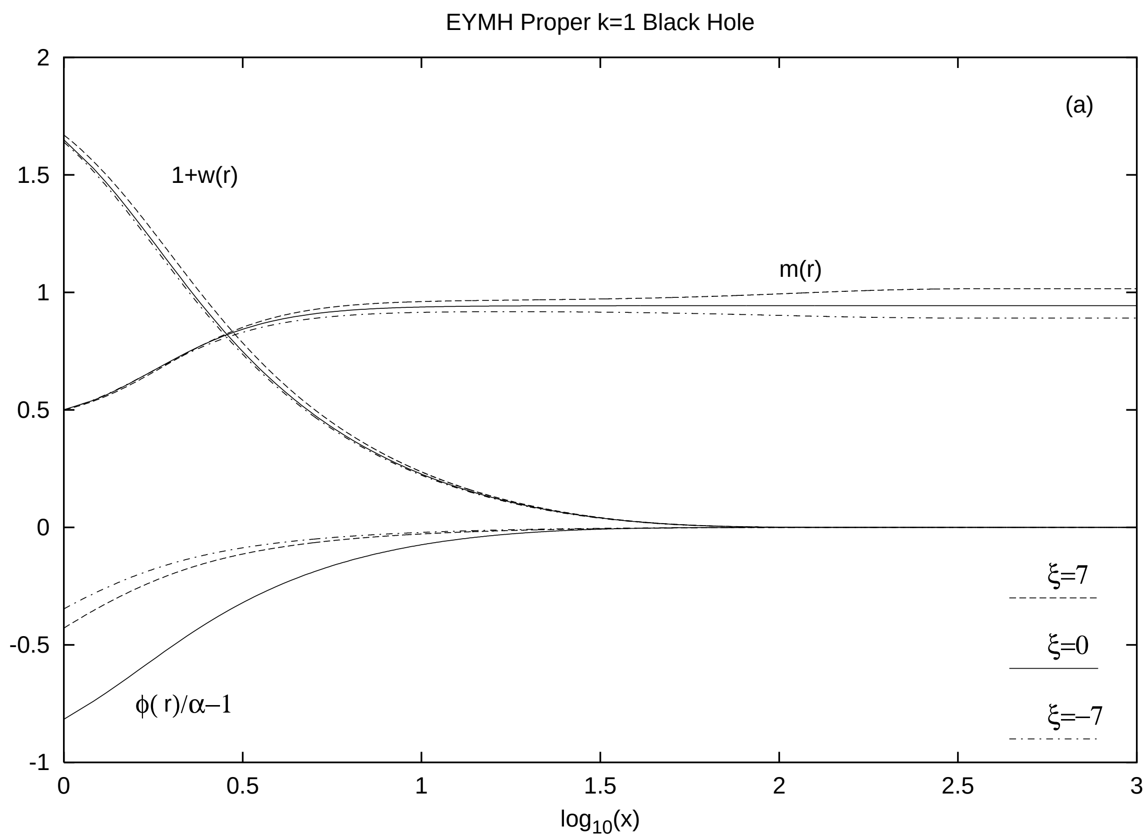


Figure 4a. Proper $k = 1$ Black Hole; $\alpha = 0.1$; $\xi = 7$, 0, -7

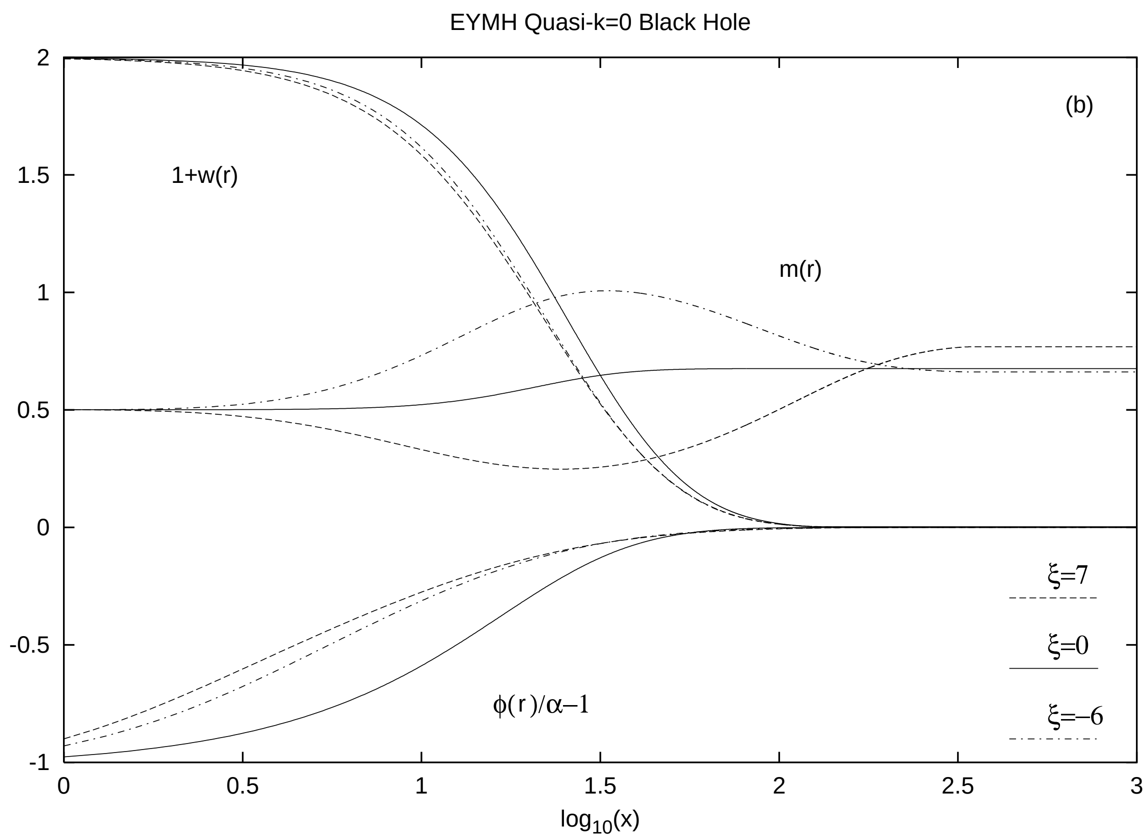


Figure 4b. Quasi$-k = 0$ Black Hole; $\alpha = 0.1$; $\xi = 7$, 0, -6

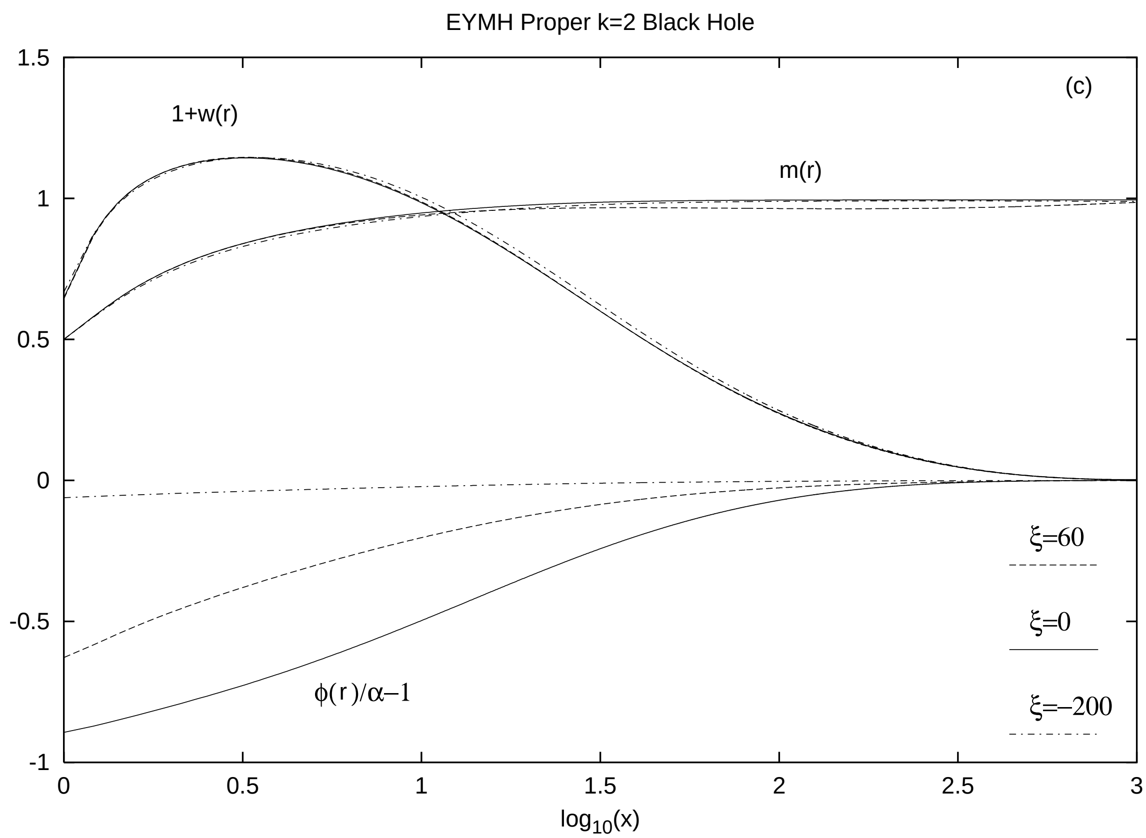


Figure 4c. Proper $k = 2$ Black Hole; $\alpha = 0.01$; $\xi = 60$, 0, -200

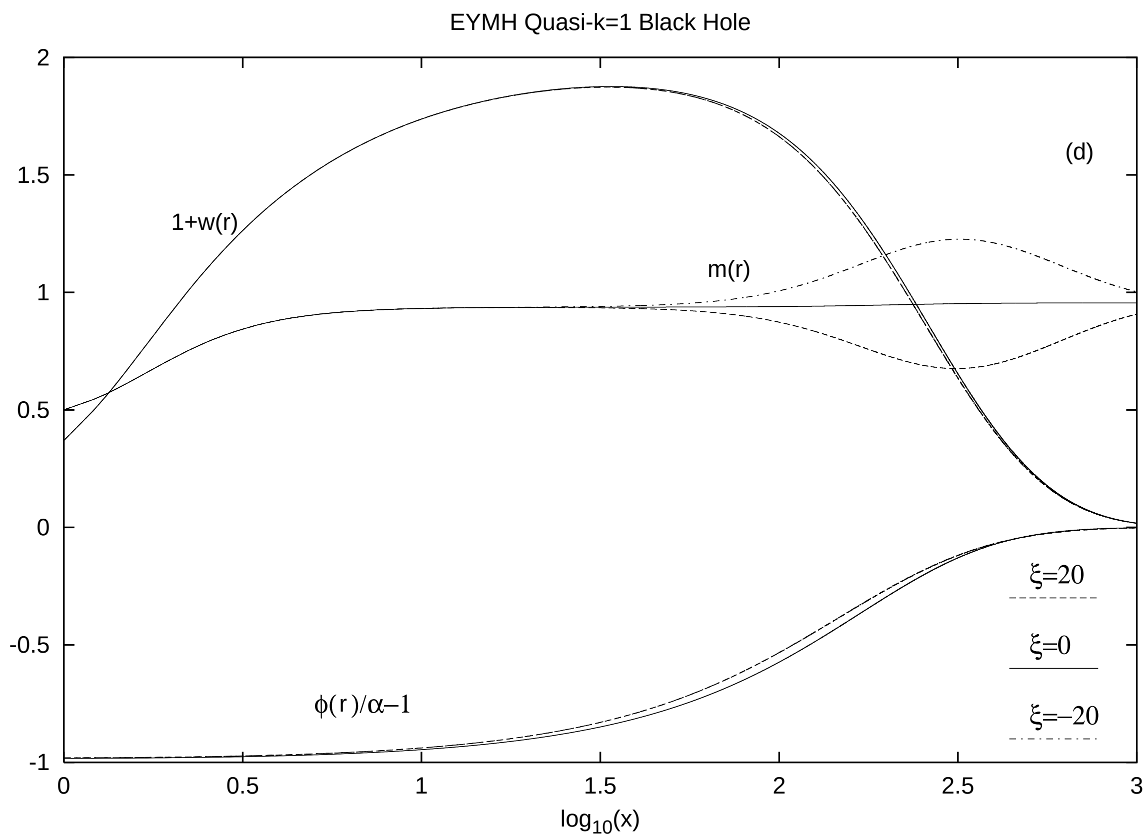


Figure 4d. Quasi$-k = 1$ Black Hole; $\alpha = 0.01$; $\xi = 20$, 0, -20

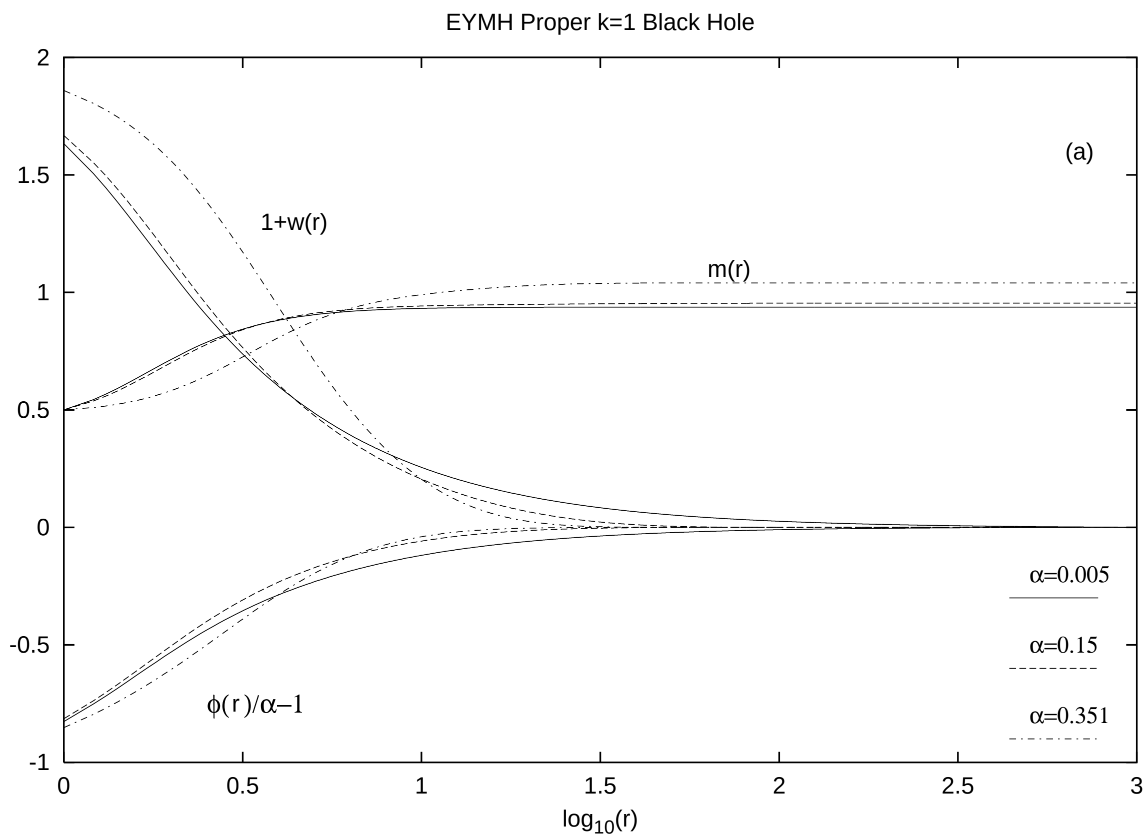


Figure 5a. Proper $k = 1$ Black Hole $\xi = 1/6$; $\alpha = 0.005,\, 0.15,\, 0.351$

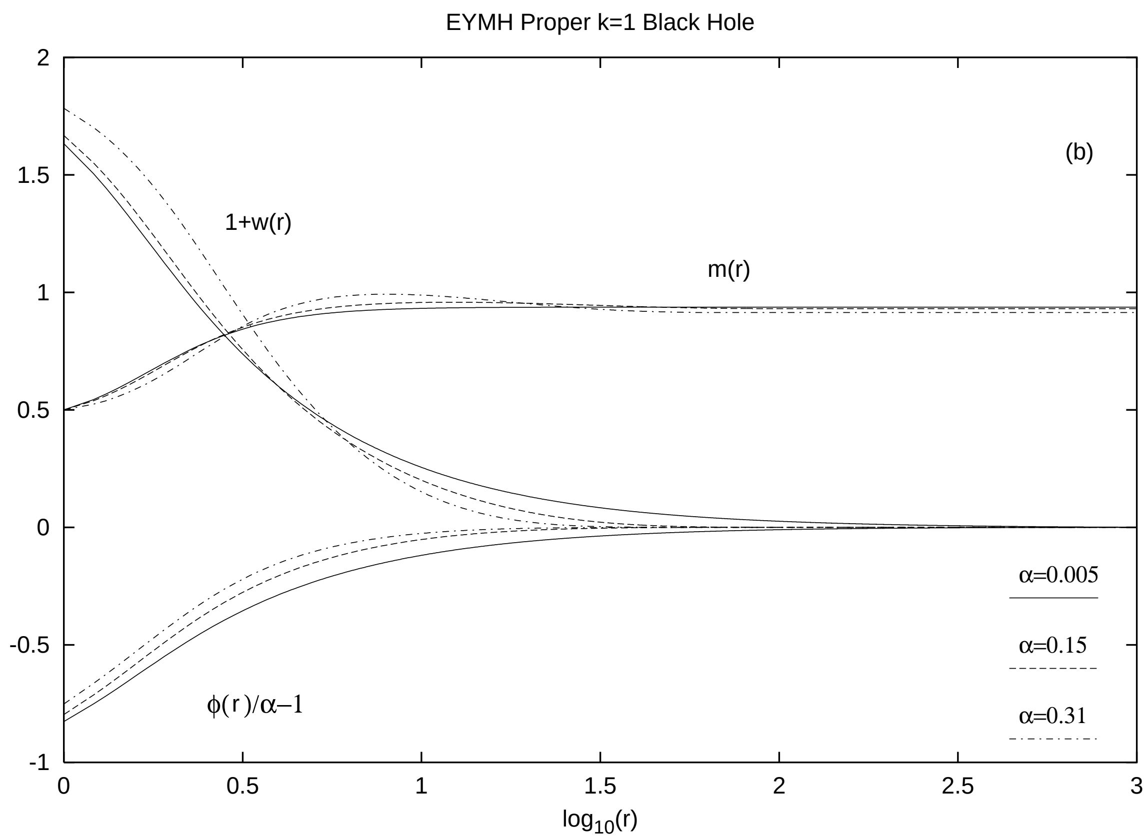


Figure 5b. Proper $k = 1$ Black Hole $\xi = -1$; $\alpha = 0.005,\, 0.15,\, 0.31$

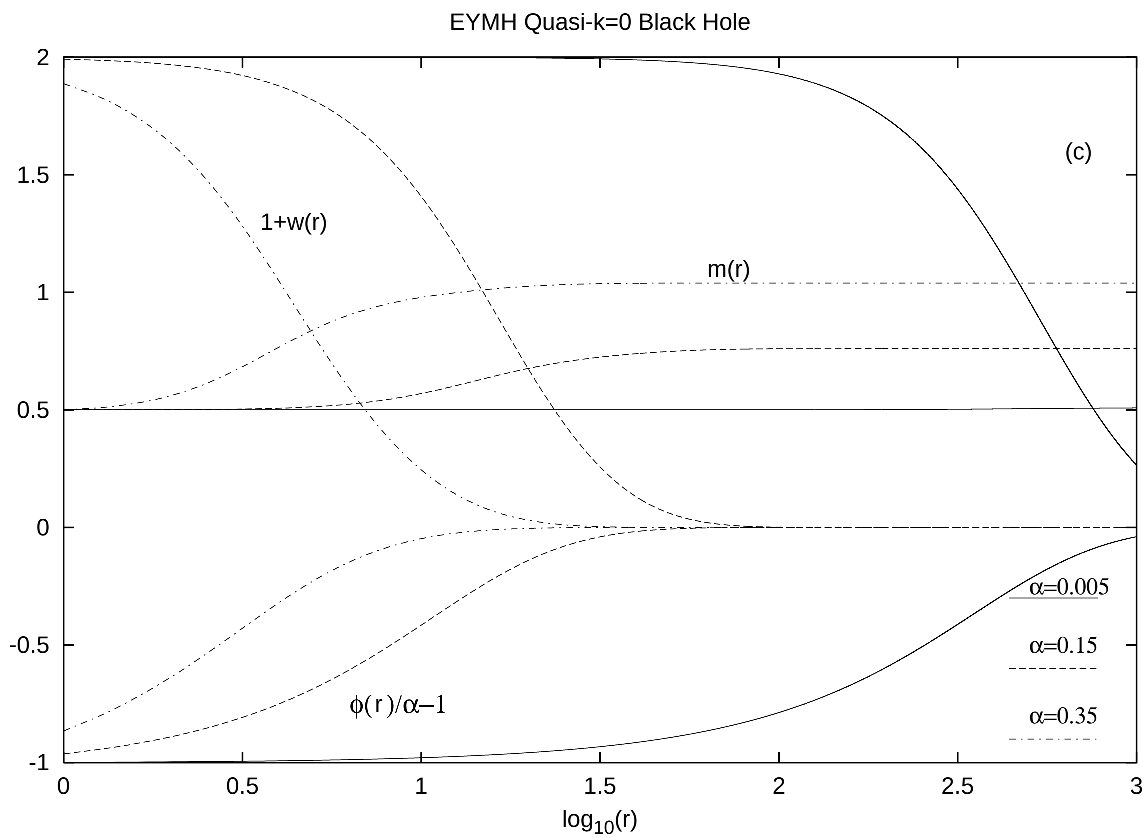


Figure 5c. Quasi$-k = 0$ Black Hole $\xi = 1/6$; $\alpha = 0.005,\, 0.15,\, 0.35$

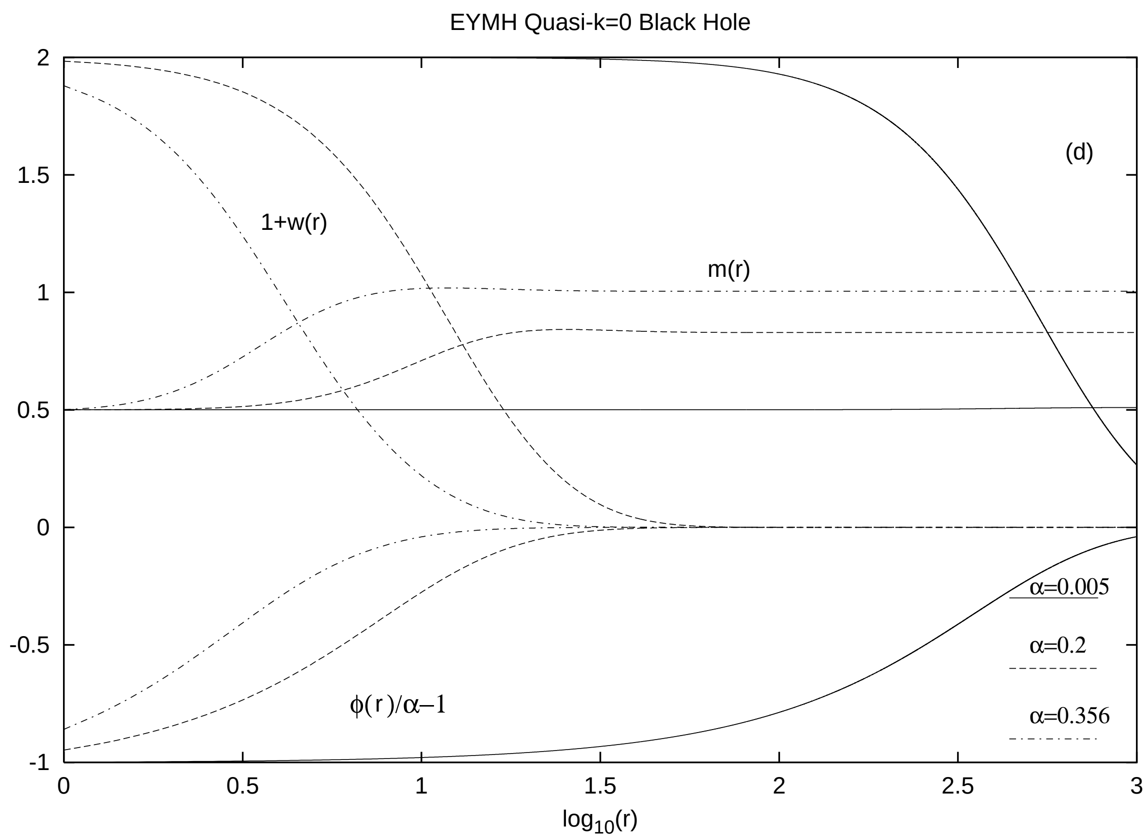


Figure 5d. Quasi$-k=0$ Black Hole $\xi=-0.1$; $\alpha=0.005,\,0.2,\,0.356$

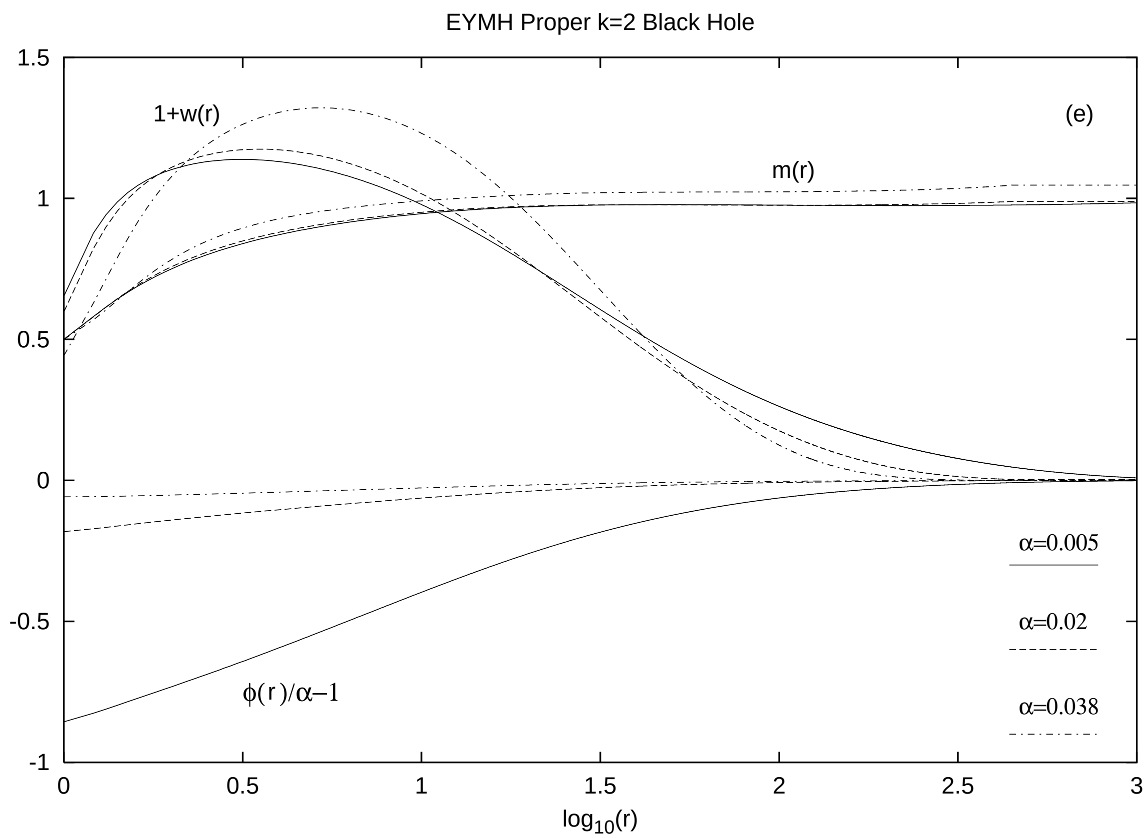


Figure 5e. Proper $k = 2$ Black Hole $\xi = 60$; $\alpha = 0.005$, 0.02, 0.038

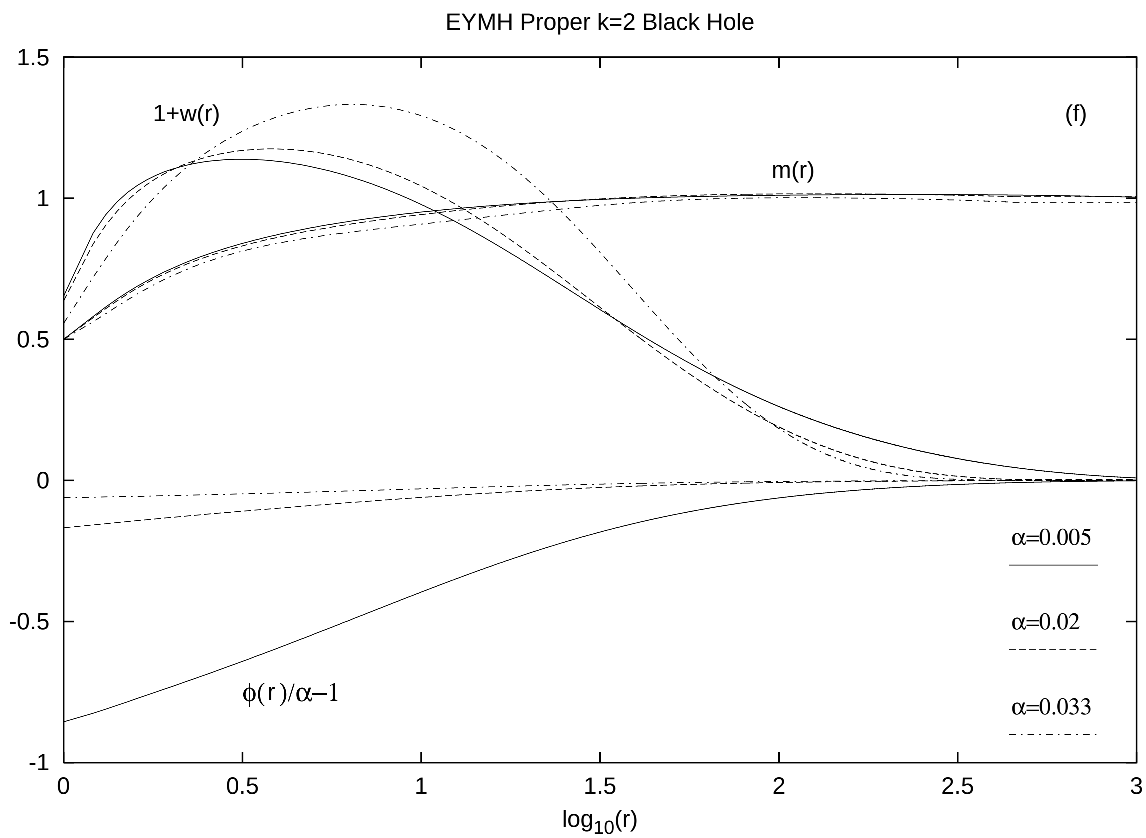


Figure 5f. Proper $k = 2$ Black Hole $\xi = -60$; $\alpha = 0.005,\, 0.02,\, 0.033$

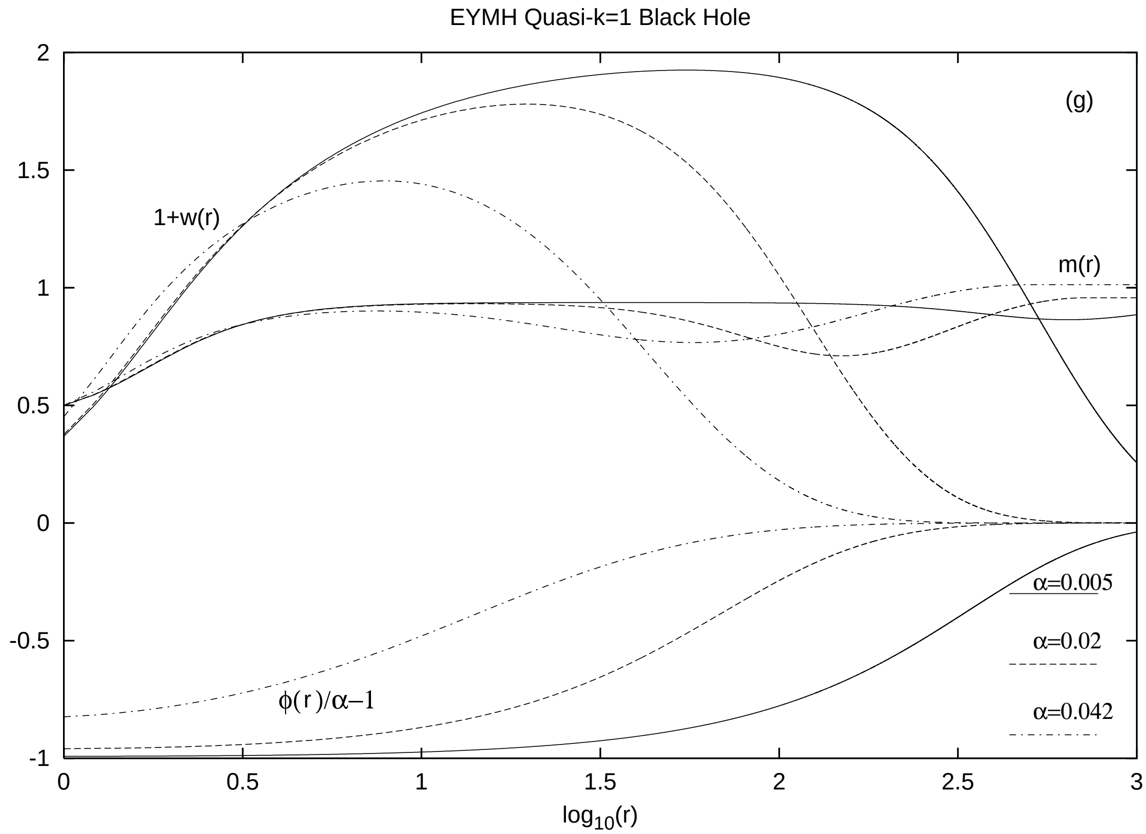


Figure 5g. Quasi-$k = 1$ Black Hole $\xi = 10$; $\alpha = 0.005$, 0.02, 0.042

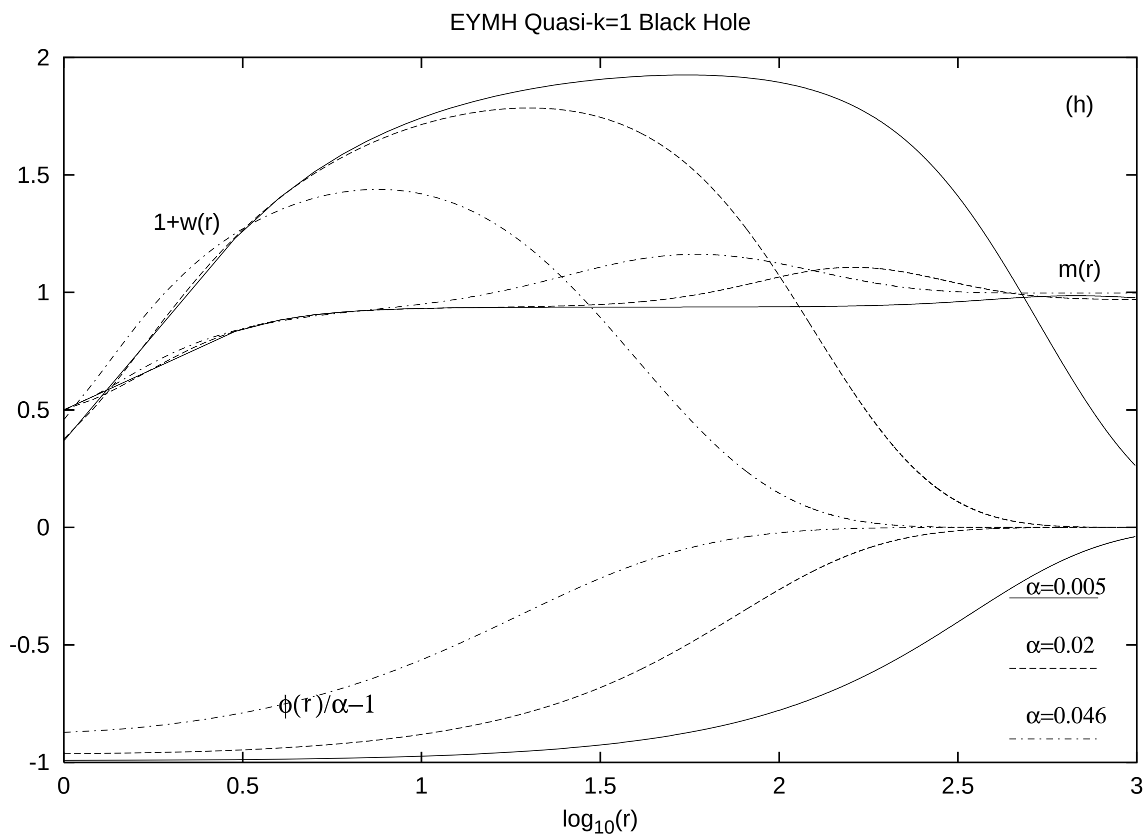


Figure 5h. Quasi-$k = 1$ Black Hole $\xi = -5$; $\alpha = 0.005$, 0.02, 0.046